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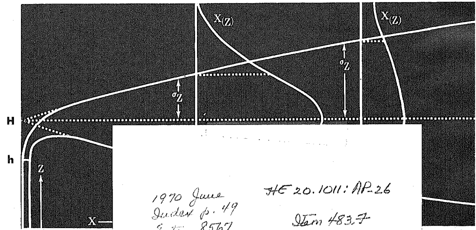
WORKBOOK OF ATMOSPHERIC DISPERSION ESTIMATES



U. S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Public Health Service
Environmental Health Service

WORKBOOK OF ATMOSPHERIC DISPERSION ESTIMATES

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Environmental Science Services Administration

U. S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Public Health Service
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PREFACE

This workbook presents some computational techniques currently used by scientists working with atmospheric dispersion problems. Because the basic working equations are general, their application to specific problems usually requires special care and judgment; such considerations are illustrated by 26 example problems. This workbook is intended as an aid to meteorologists and air pollution scientists who are required to estimate atmospheric concentrations of contaminants from various types of sources. It is not intended as a complete do-it-yourself manual for atmospheric dispersion estimates; all of the numerous complications that arise in making best estimates of dispersion cannot be so easily resolved. Awareness of the possible complexities can enable the user to appreciate the validity of his "first approximations" and to realize when the services of a professional air pollution meteorologist are required.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Robert A. McCormick, Paul A. Humphrey, and other members of the Field Research Office for their helpful discussions and review; to Jean J. Schueneman, Chief, Criteria and Standards Development, National Center for Air Pollution Control, who suggested this workbook; to Phyllis Polland and Frank Schiermeier, who checked the problem solutions; to Ruth Umfleet and Edna Beasley for their aid; and to the National Center for Air Pollution Control, Public Health Service, and Air Resources Laboratory, Environmental Science Services Administration, for their support.

CONTENTS

ABSTRACT

This workbook presents methods of practical application of the binormal continuous plume dispersion model to estimate concentrations of air pollutants. Estimates of dispersion are those of Pasquill as restated by Gifford. Emphasis is on the estimation of concentrations from continuous sources for sampling times up to 1 hour. Some of the topics discussed are determination of effective height of emission, extension of concentration estimates to longer sampling intervals, inversion break-up fumigation concentrations, and concentrations from area, line, and multiple sources. Twenty-six example problems and their solutions are given. Some graphical aids to computation are included.

Chapter 1 — INTRODUCTION

During recent years methods of estimating atmospheric dispersion have undergone considerable revision, primarily due to results of experimental measurements. In most dispersion problems the relevant atmospheric layer is that nearest the ground, varying in thickness from several hundred to a few thousand meters. Variations in both thermal and mechanical turbulence and in wind velocity are greatest in the layer in contact with the surface. Turbulence induced by buoyancy forces in the atmosphere is closely related to the vertical temperature structure. When temperature decreases with height at a rate higher than 5.4°F per 1000 ft (1°C per 100 meters), the atmosphere is in unstable equilibrium and vertical motions are enhanced. When temperature decreases at a lower rate or increases with height (inversion), vertical motions are damped or reduced. Examples of typical variations in temperature and wind speed with height for daytime and nighttime conditions are illustrated in Figure 1-1.

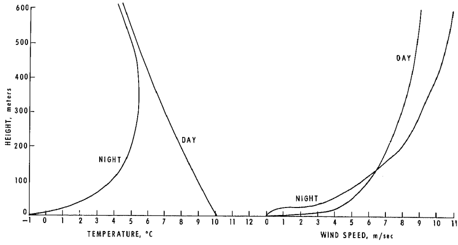

Figure 1-1. Examples of variation of temperature and wind speed with height (after Smith, 1963).

The transfer of momentum upward or downward in the atmosphere is also related to stability; when the atmosphere is unstable, usually in the daytime, upward motions transfer the momentum "deficiency" due to eddy friction losses near the earth's surface through a relatively deep layer, causing the wind speed to increase more slowly with height than at night (except in the lowest few meters). In addition to thermal turbulence, roughness elements on the ground engender mechanical turbulence, which affects both the dispersion of material in the atmosphere and the wind profile (variation of wind with height). Examples of these effects on the resulting wind profile are shown in Figure 1-2.

As wind speed increases, the effluent from a continuous source is introduced into a greater volume of air per unit time interval. In addition to this dilution by wind speed, the spreading of the material (normal to the mean direction of transport) by turbulence is a major factor in the dispersion process.

The procedures presented here to estimate atmospheric dispersion are applicable when mean wind speed and direction can be determined, but measurements of turbulence, such as the standard deviation of wind direction fluctuations, are not available. If such measurements are at hand, techniques such as those outlined by Pasquill (1961) are likely to give more accurate results. The diffusion parameters presented here are most applicable to ground-level or low-level releases (from the surface to about 20 meters), although they are commonly applied at higher elevations without full experimental validation. It is assumed that stability is the same throughout the diffusing layer, and no turbulent transfer occurs *through* layers of dissimilar stability characteristics. Because mean values for wind directions and speeds are required, neither the variation of wind speed nor the variation of wind direction with height in the mixing layer are taken into account. This usually is not a problem in neutral or unstable (e.g., daytime) situations, but can cause over-estimations of downwind concentrations in stable conditions.

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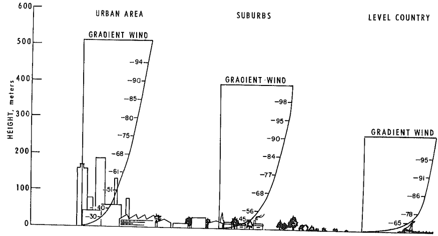

Figure 1-2. Examples of variation of wind with height over different size roughness elements (figures are percentages of gradient wind); (from Davenport, 1963).

Chapter 2 — BACKGROUND

For a number of years estimates of concentrations were calculated either from the equations of Sutton (1932) with the atmospheric dispersion parameters C_y, C_z, and n, or from the equations of Bosanquet (1936) with the dispersion parameters p and q.

Hay and Pasquill (1957) have presented experimental evidence that the vertical distribution of spreading particles from an elevated point is related to the standard deviation of the wind elevation angle, σ_E, at the point of release. Cramer (1957) derived a diffusion equation incorporating standard deviations of Gaussian distributions: σ_y for the distribution of material in the plume across wind in the horizontal, and σ_z for the vertical distribution of material in the plume. (See Appendix 2 for properties of Gaussian distributions.) These statistics were related to the standard deviations of azimuth angle, σ_A, and elevation angle, σ_E, calculated from wind measurements made with a bi-directional wind vane (bivane). Values for diffusion parameters based on field diffusion tests were suggested by Cramer, et al. (1958) (and also in Cramer 1959a and 1959b). Hay and Pasquill (1959) also presented a method for deriving the spread of pollutants from records of wind fluctuation. Pasquill (1961) has further proposed a method for estimating diffusion when such detailed wind data are not available. This method expresses the height and angular spread of a diffusing plume in terms of more commonly observed weather parameters. Suggested curves of height and angular spread as a function of distance downwind were given for several "stability" classes. Gifford (1961) converted Pasquill's values of angular spread and height into standard deviations of plume concentration distribution, σ_y and σ_z. Pasquill's method, with Gifford's conversion incorporated, is used in this workbook (see Chapter 3) for diffusion estimates.

Advantages of this system are that (1) only two dispersion parameters are required and (2) results of most diffusion experiments are now being reported in terms of the standard deviations of plume spread. More field dispersion experiments are being conducted and will be conducted under conditions of varying surface roughness and atmospheric stability. If the dispersion parameters from a specific experiment are considered to be more representative than those suggested in this workbook, the parameter values can be used with the equations given here.

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is chapter outlines the basic procedures to ·d in making dispersion estimates as sug- by Pasquill (1961) and modified by Gifford .

DINATE SYSTEM

the system considered here the origin is at l level at or beneath the point of emission, ıe x-axis extending horizontally in the direc- the mean wind. The y-axis is in the hori- plane perpendicular to the x-axis, and the extends vertically. The plume travels along ıllel to the x-axis. Figure 3-1 illustrates the ıate system.

SION EQUATIONS

ə concentration, χ, of gas or aerosols (parti- ss than about 20 microns diameter) at x,y,z continuous source with an effective emission H, is given by equation 3.1. The notation o depict this concentration is χ (x,y,z;H). he height of the plume centerline when it becomes essentially level, and is the sum of the physical stack height, h, and the plume rise, ΔH. The following assumptions are made: the plume spread has a Gaussian distribution (see Appendix 2) in both the horizontal and vertical planes, with standard deviations of plume concentration distribution in the horizontal and vertical of σ_y and σ_z, respectively; the mean wind speed affecting the plume is u; the uniform emission rate of pollutants is Q; and total reflection of the plume takes place at the earth's surface, i.e., there is no deposition or reaction at the surface (see problem **9**).

$$\chi\,(x,y,z;H) = \frac{Q}{2\pi\,\sigma_y\,\sigma_z\,u}\,\exp^{*}\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right]+\exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right]\right\} \qquad (3.1)$$

*Note: exp $-a/b = e^{-a/b}$ where e is the base of natural logarithms and is approximately equal to 2.7183.

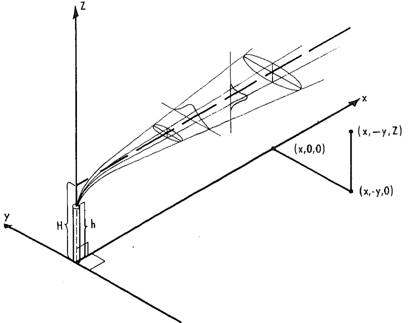

Figure 3-1. Coordinate system showing Gaussian distributions in the horizontal and vertical.

Any consistent set of units may be used. The most common is:

χ (g m^{-3}) or, for radioactivity (curies m^{-3})
Q (g sec^{-1}) or (curies sec^{-1})
u (m sec^{-1})
σ_y, σ_z, H,x,y, and z (m)

This equation is the same as equation (8.35) p. 293 of Sutton (1953) when σ's are substituted for Sutton's parameters through equations like (8.27) p. 286. For evaluations of the exponentials found in Eq. (3.1) and those that follow, see Appendix 3. χ is a mean over the same time interval as the time interval for which the σ's and u are representative. The values of both σ_y and σ_z are evaluated in terms of the downwind distance, x.

Eq. (3.1) is valid where diffusion in the direction of the plume travel can be neglected, that is, no diffusion in the x direction.

This may be assumed if the release is continuous or if the duration of release is equal to or greater than the travel time (x/u) from the source to the location of interest.

For concentrations calculated at ground level, i.e., z = 0, (see problem 3) the equation simplifies to:

$$\chi\,(x,y,0;H) = \frac{Q}{\pi\,\sigma_y\,\sigma_z\,u}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \quad (3.2)$$

Where the concentration is to be calculated along the centerline of the plume (y = 0), (see problem 2) further simplification results:

$$\chi\,(x,0,0;H) = \frac{Q}{\pi\,\sigma_y\,\sigma_z\,u}\exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \quad (3.3)$$

For a ground-level source with no effective plume rise (H = 0), (see problem 1):

$$\chi\,(x,0,0;0) = \frac{Q}{\pi\,\sigma_y\,\sigma_z\,u} \quad (3.4)$$

EFFECTS OF STABILITY

The values of σ_y and σ_z vary with the turbulent structure of the atmosphere, height above the surface, surface roughness, sampling time over which the concentration is to be estimated, wind speed, and distance from the source. For the parameter values given here, the sampling time is assumed to be about 10 minutes, the height to be the lowest several hundred meters of the atmosphere, and the surface to be relatively open country. The turbulent structure of the atmosphere and wind speed are considered in the stability classes presented, and the effect of distance from the source is considered in the graphs determining the parameter values. Values for σ_y and σ_z are estimated from the stability of the atmosphere, which is in turn estimated from the wind speed at a height of about 10 meters and, during the day, the incoming solar radiation or, during the night, the cloud cover (Pasquill, 1961). Stability categories (in six classes) are given in Table 3-1. Class A is the most unstable, class F the most stable class considered here. Night refers to the period from 1 hour before sunset to 1 hour after sunrise. Note that the neutral class, D, can be assumed for overcast conditions during day or night, regardless of wind speed.

Table 3-1 KEY TO STABILITY CATEGORIES

Surface Wind Speed (at 10 m), m sec^{-1}	Day: Incoming Solar Radiation			Night	
	Strong	Moderate	Slight	Thinly Overcast or $\geq$4/8 Low Cloud	$\leq$3/8 Cloud
< 2	A	A-B	B		
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
> 6	C	D	D	D	D

The neutral class, D, should be assumed for overcast conditions during day or night.

"Strong" incoming solar radiation corresponds to a solar altitude greater than 60° with clear skies; "slight" insolation corresponds to a solar altitude from 15° to 35° with clear skies. Table 170, Solar Altitude and Azimuth, in the Smithsonian Meteorological Tables (List, 1951) can be used in determining the solar altitude. Cloudiness will decrease incoming solar radiation and should be considered along with solar altitude in determining solar radiation. Incoming radiation that would be strong with clear skies can be expected to be reduced to moderate with broken (5/8 to 7/8 cloud cover) middle clouds and to slight with broken low clouds. An objective system of classifying stability from hourly meteorological observations based on the above method has been suggested (Turner, 1961).

These methods will give representative indications of stability over open country or rural areas, but are less reliable for urban areas. This difference is due primarily to the influence of the city's larger surface roughness and heat island effects upon the stability regime over urban areas. The greatest difference occurs on calm clear nights; on such nights conditions over rural areas are very stable, but over urban areas they are slightly unstable or near neutral to a height several times the average building height, with a stable layer above (Duckworth and Sandberg, 1954; DeMarrais, 1961).

Some preliminary results of a dispersion experiment in St. Louis (Pooler, 1965) showed that the dispersion over the city during the daytime behaved somewhat like types B and C; for one night experiment σ_y varied with distance between types D and E.

ESTIMATION OF VERTICAL AND HORIZONTAL DISPERSION

Having determined the stability class from Table 3-1, one can evaluate the estimates of σ_y and σ_z as a function of downwind distance from the source, x, using Figures 3-2 and 3-3. These values of σ_y and σ_z are representative for a sampling time of about 10 minutes. For estimation of concentrations for longer time periods see Chapter 5. Figures 3-2 and 3-3 apply strictly only to open level country and probably underestimate the plume dispersion potential from low-level sources in built-up areas. Although the vertical spread may be less than the values for class F with very light winds on a clear night, quantitative estimates of concentrations are nearly impossible for this condition. With very light winds on a clear night for ground-level sources free of topographic influences, frequent shifts in wind direction usually occur which serve to spread the plume horizontally. For elevated sources under these extremely stable situations, significant concentrations usually do not reach ground level until the stability changes.

A stable layer existing above an unstable layer will have the effect of restricting the vertical diffusion. The dispersion computation can be modified for this situation by considering the height of the base of the stable layer, L. At a height 2.15 σ_z above the plume centerline the concentration is one-tenth the plume centerline concentration at the same distance. When one-tenth the plume centerline concentration extends to the stable layer, at height L, it is reasonable to assume that the distribution starts being affected by the "lid." The following method is suggested to take care of this situation. Allow σ_z to increase with distance to a value of L/2.15 or 0.47 L. At this distance x_L, the plume is assumed to have a Gaussian distribution in the vertical. Assume that by the time the plume travels twice this far, 2 x_L, the plume has become uniformly distributed between the earth's surface and the height L, i.e., concentration does not vary with height (see Figure 3-4). For the distances greater than 2 x_L, the concentration for any height between the ground and L can be calculated from:

$$\chi\,(x,y,z;H) = \frac{Q}{\sqrt{2\pi}\,\sigma_y\,L\,u}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \quad (3.5)$$

for any z from 0 to L
for x > 2 x_L; x_L is where $\sigma_z = 0.47$ L

(see problem 6). Note that Eq. (3.5) assumes normal or Gaussian distribution of the plume *only* in the horizontal plane. The same result can be obtained from the following equation where σ_{zL} is an effective dispersion parameter because $\sqrt{2\pi}$ L = 2.5066 L and 0.8 πL = 2.51 L.

$$\chi\,(x,y,z;H) = \frac{Q}{\pi\,\sigma_y\,\sigma_{zL}\,u}\left[\exp -\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \quad (3.6)$$

for any z from 0 to L
for x > 2_{xL}; x_L is where $\sigma_z = 0.47$ L
The value of $\sigma_{zL} = 0.8$ L

EVALUATION OF WIND SPEED

For the wind speed, u, a mean through the vertical extent of the plume should be used. This would be from the height H − 2 σ_z through H + 2 σ_z. Of course, if 2 σ_z is greater than H then the wind can be averaged from the ground to H + 2 σ_z. However, the "surface wind" value may be all that is available. The surface wind is most applicable to surface or low-level emissions, especially under stable conditions.

PLOTS OF CONCENTRATIONS AGAINST DISTANCE

To gain maximum insight into a diffusion problem it is often desirable to plot centerline concentrations against distance downwind. A convenient procedure is to determine the ground-level centerline concentrations for a number of downwind distances and plot these values on log-log graph paper. By connecting the points, one may estimate concentrations for intermediate downwind distances (see problem 6).

ACCURACY OF ESTIMATES

Because of a multitude of scientific and technical limitations the diffusion computation method presented in this manual may provide *best estimates* but not infallible predictions. In the unstable and stable cases, severalfold errors in estimate of σ_z can occur for the longer travel distances. In some cases the σ_z may be expected to be correct within a factor of 2, however. These are: (1) all stabilities for distance of travel out to a few hundred meters; (2) neutral to moderately unstable conditions for distances out to a few kilometers; and (3) unstable conditions in the lower 1000 meters of the atmosphere with a marked inversion above for distances out to 10 km or more. Uncertainties in the estimates of σ_y are in general less than those of σ_z. The ground-level centerline concentrations for these

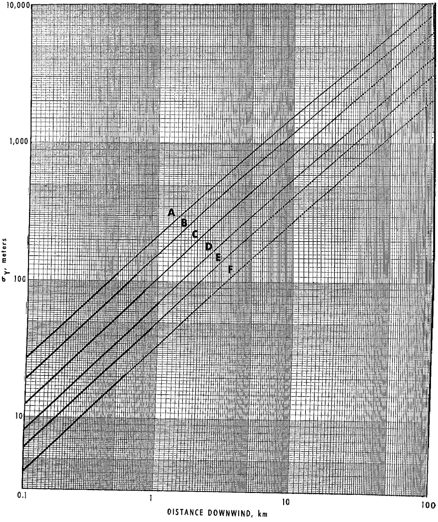

Figure 3-2. Horizontal dispersion coefficient as a function of downwind distance from the source.

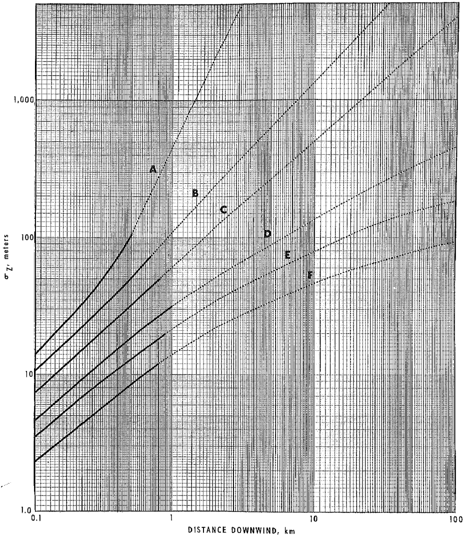

Figure 3-3. Vertical dispersion coefficient as a function of downwind distance from the source.

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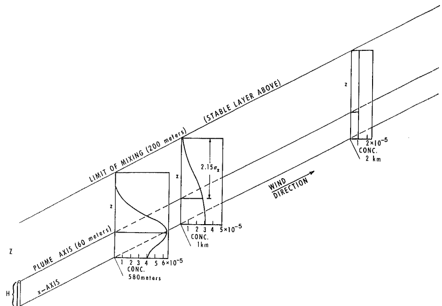

Figure 3-4. Variations in concentration in the vertical beneath a more stable layer.

three cases (where σ_z can be expected to be within a factor of 2) should be correct within a factor of 3, including errors in σ_y and u. The relative confidence in the σ's (in decreasing order) is indicated by the heavy lines and dashed lines in Figures 3-2 and 3-3.

Estimates of H, the effective height of the plume, may be in error because of uncertainties in the estimation of ΔH, the plume rise. Also, for problems that require estimates of concentration at a specific point, the difficulty of determining the mean wind over a given time interval and consequently the location of the x-axis can cause considerable uncertainty.

GRAPHS FOR ESTIMATES OF DIFFUSION

To avoid repetitious computations, Figure 3-5 (A through F) gives relative ground-level concentrations times wind speed (χ u/Q) against downwind distances for various effective heights of emission and limits to the vertical mixing for each stability class (1 figure for each stability). Computations were made from Eq. (3.3), (3.4), and (3.5). Estimates of actual concentrations may be determined by multiplying ordinate values by Q/u.

PLOTTING GROUND-LEVEL CONCENTRATION ISOPLETHS

Often one wishes to determine the locations where concentrations equal or exceed a given magnitude. First, the axial position of the plume must be determined by the mean wind direction. For plotting isopleths of ground-level concentrations, the relationship between ground-level centerline concentrations and ground-level off-axis concentrations can be used:

$$\frac{\chi\ (x,y,0;H)}{\chi\ (x,0,0;H)} = \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \quad (3.7)$$

The y coordinate of a particular isopleth from the x-axis can be determined at each downwind distance, x. Suppose that one wishes to know the off-axis distance to the 10^{-3} g m^{-3} isopleth at an x of 600 m, under stability type B, where the ground-level centerline concentration at this distance is 2.9 x 10^{-3} g m^{-3}.

$$\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] = \frac{\chi\ (x,y,0;H)}{\chi\ (x,0,0;H)} =$$

$$\frac{10^{-3}}{2.9 \times 10^{-3}} = 0.345$$

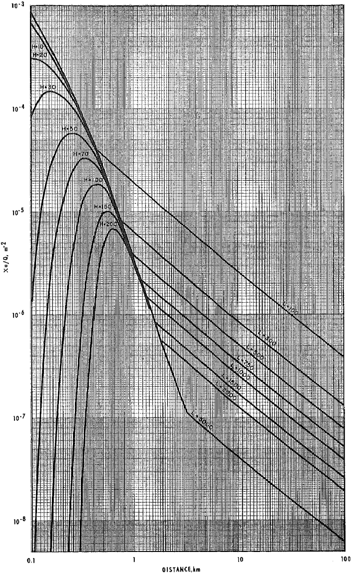

Figure 3-5A. χu/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), A stability.

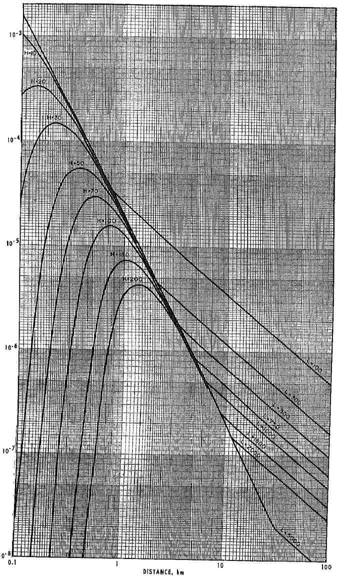

Figure 3-5B. χu/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), B stability.

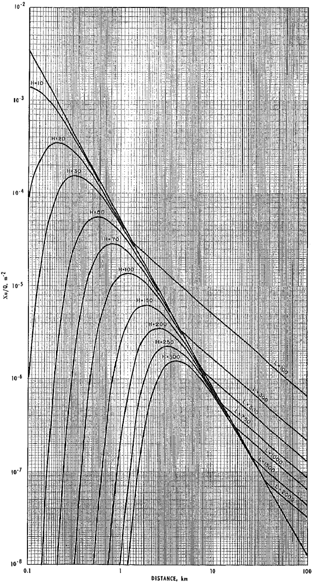

Figure 3-5C. $\chi u/Q$ with distance for various heights of emission (H) and limits to vertical dispersion (L), C stability.

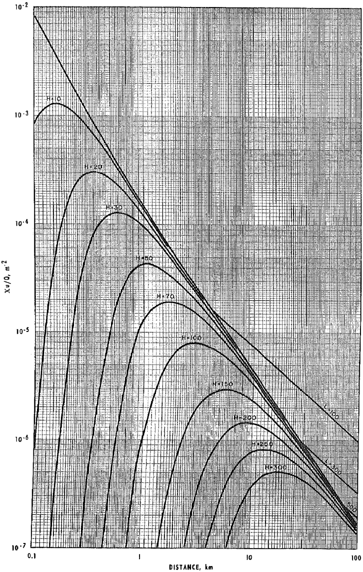

Figure 3-5D. χu/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), D stability.

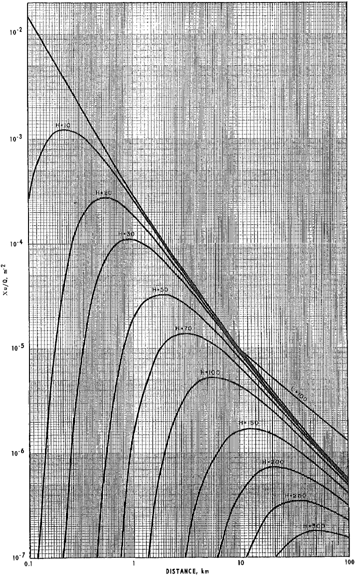

Figure 3-5E. χu/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), E stability.

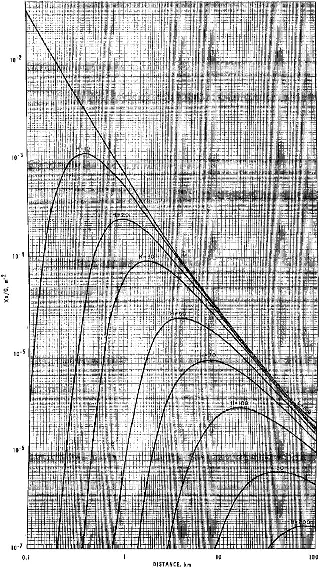

Figure 3-5F. χu/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), F stability.

From Table A-1 (Appendix 3) when exp $\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] = 0.345, y/\sigma_y = 1.46$

From Figure 3-2, for stability B and x = 600 m, $\sigma_y = 92$. Therefore y = (1.46) (92) = 134 meters. This is the distance of the 10^{-3} isopleth from the x-axis at a downwind distance of 600 meters.

This can also be determined from:

$$y = \left\{2 \ln^* \left[\frac{\chi\,(x,0,0;H)}{\chi\,(x,y,0;H)}\right]\right\}^{1/2} \sigma_y \qquad (3.8)$$

The position corresponding to the downwind distance and off-axis distance can then be plotted. After a number of points have been plotted, the concentration isopleth may be drawn (see problems 8 and 26). Figures 3-6 and 3-7 give ground-level isopleths of $\chi u/Q$ for various stabilities for sources at H = 0 and H = 100 meters. For example, to locate the 10^{-3} g m^{-3} isopleth resulting from a ground-level source of 20 g sec^{-1} under B stability conditions with wind speed 2 m sec^{-1}, one must first determine the corresponding value of $\chi u/Q$ since this is the quantity graphed in Figure 3-6. $\chi u/Q = 10^{-3} \times 2/20 = 10^{-4}$. Therefore the $\chi u/Q$ isopleth in Figure 3-6B having a value of 10^{-4} m^{-2} corresponds to a χ isopleth with a value of 10^{-3} g m^{-3}.

AREAS WITHIN ISOPLETHS

Figure 3-8 gives areas within isopleths of ground-level concentration in terms of χ u/Q for a ground-level source for various stability categories (Gifford, 1962; Hilsmeier and Gifford, 1962). For the example just given, the area of the 10^{-3} g m^{-3} isopleth (10^{-4} m^{-2} χ u/Q isopleth) is about 5 x 10^4 meter^2.

CALCULATION OF MAXIMUM GROUND-LEVEL CONCENTRATIONS

Figure 3-9 gives the distance to the point of maximum concentration, x_{max}, and the relative maximum concentration, χ u/Q_{max}, as a function of effective height of emission and stability class (Martin, 1965). This figure was prepared from graphs of concentration versus distance, as in Figure 3-5. The maximum concentration can be determined by finding χ u/Q as a function of effective emission height and stability and multiplying by Q/u. In using Figure 3-9, the user must keep in mind that the dispersion at higher levels may differ considerably from that determined by the σ_y's and σ_z's used here. As noted, however, since σ_y generally decreases with height and u increases with height, the product u σ_y σ_z will not change appreciably. The greater the effective height, the more likely it is that the stability may not be the same from the ground to this height. With the longer travel distances such as the points of maximum concentrations for stable conditions (Types E or F), the stability may change before the plume travels the entire distance.

*"ln" denotes natural logarithms, i.e., to the base e.

REVIEW OF ASSUMPTIONS

The preceding has been based on these assumptions, which should be clearly understood:

(i) Continuous emission from the source or emission times equal to or greater than travel times to the downwind position under consideration, so that diffusion in the direction of transport may be neglected.

(ii) The material diffused is a stable gas or aerosol (less than 20 microns diameter) which remains suspended in the air over long periods of time.

(iii) The equation of continuity:

$$Q = \int_0^{+\infty} \int_{-\infty}^{+\infty} \chi\, u\, dy\, dz \qquad (3.9)$$

is fulfilled, i.e., none of the material emitted is removed from the plume as it moves downwind and there is complete reflection at the ground.

(iv) The mean wind direction specifies the x-axis, and a mean wind speed representative of the diffusing layer is chosen.

(v) Except where specifically mentioned, the plume constituents are distributed normally in both the cross-wind and vertical directions.

(vi) The σ's given in Figures 3-2 and 3-3 represent time periods of about 10 minutes.

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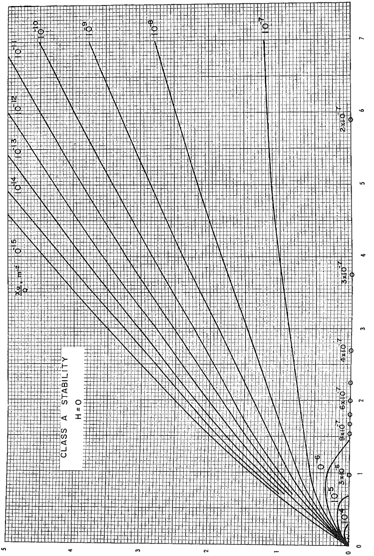

Figure 3-6A. Isopleths of χu/Q for a ground-level source, A stability.

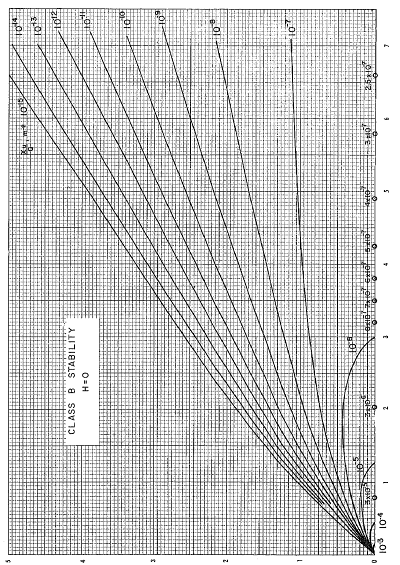

Figure 3-6B. Isopleths of χu/Q for a ground-level source, B stability.

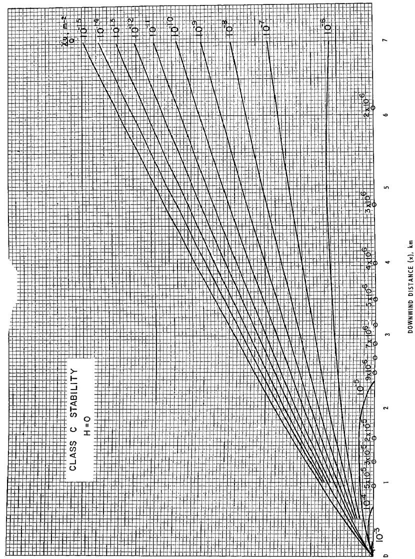

Figure 3-6C. Isopleths of $\chi u/Q$ for a ground-level source, C stability.

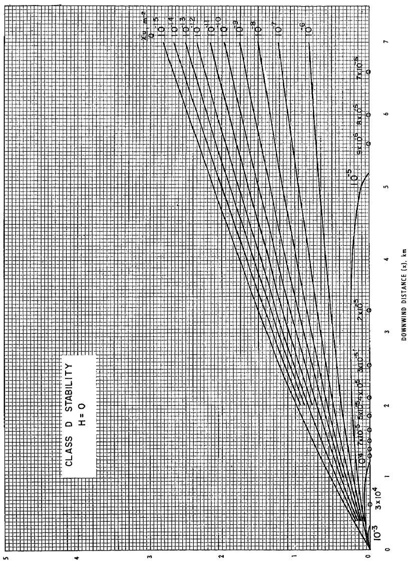

Figure 3-6D. Isopleths of χu/Q for a ground-level source, D stability.

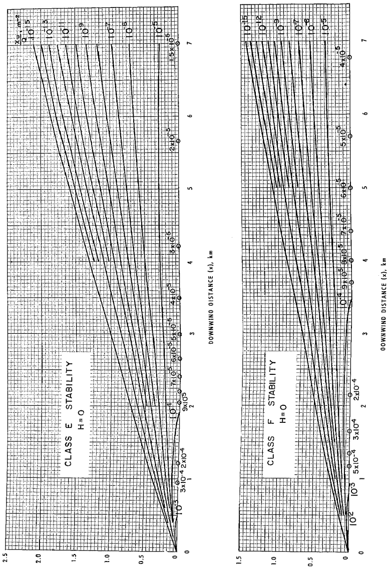

Figure 3-6E, F. Isopleths of $\chi u/Q$ for a ground-level source, E and F stabilities.

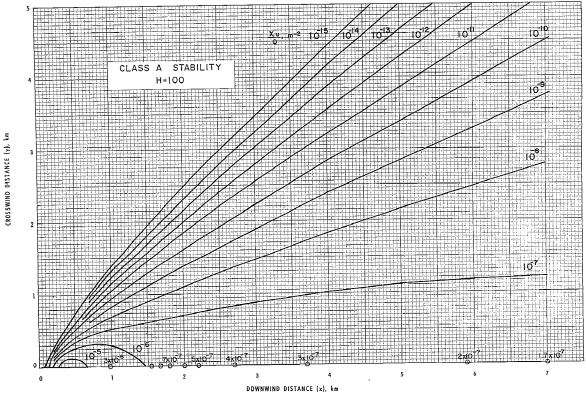

Figure 3-7A. Isopleths of χu/Q for a source 100 meters high, A stability.

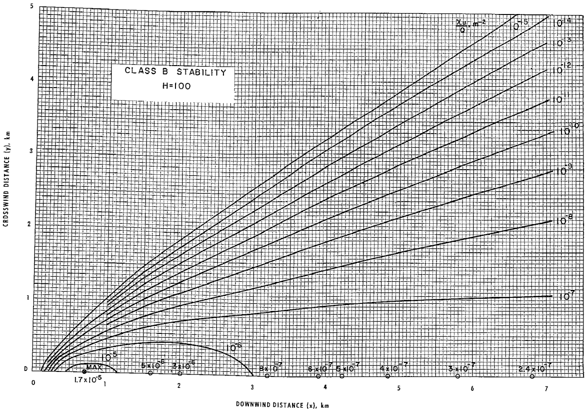

Figure 3-7B. Isopleths of $\chi u/Q$ for a source 100 meters high, B stability.

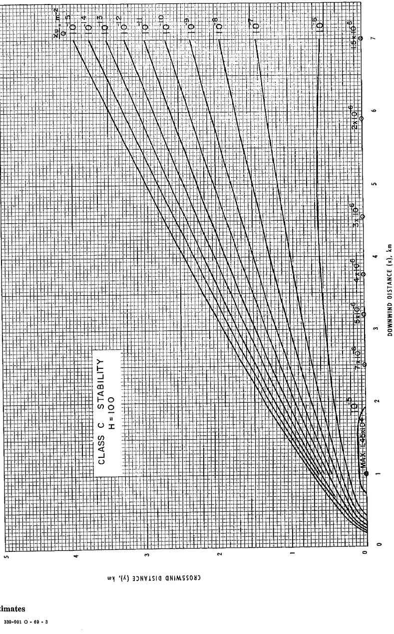

Figure 3-7C. Isopleths of χu/Q for a source 100 meters high, C stability.

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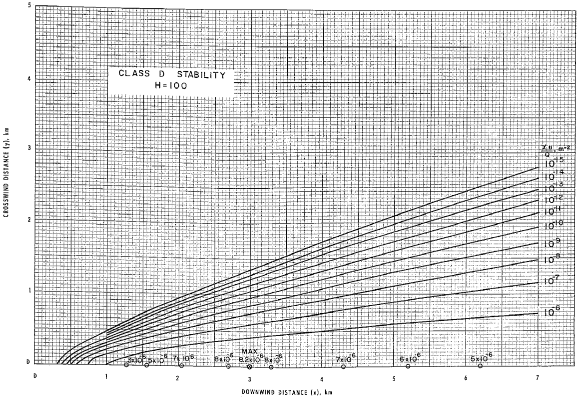

Figure 3-7D. Isopleths of $\chi u/Q$ for a source 100 meters high, D stability.

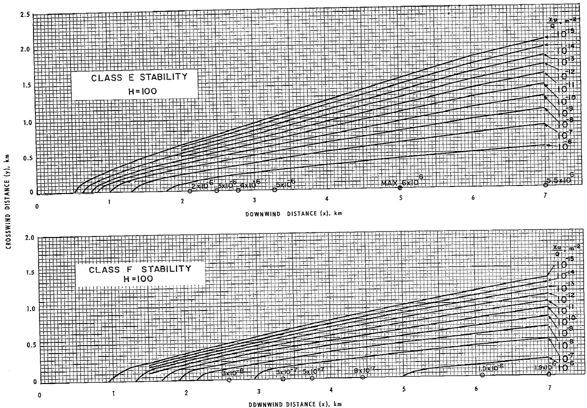

Figure 3-7E, F. Isopleths of χu/Q for a source 100 meters high, E and F stabilities.

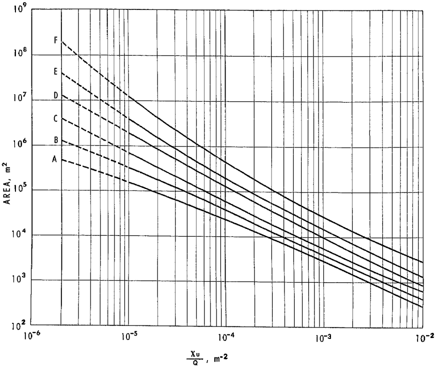

Figure 3-8. Area within isopleths for a ground-level source (from Hilsmeier and Gifford).

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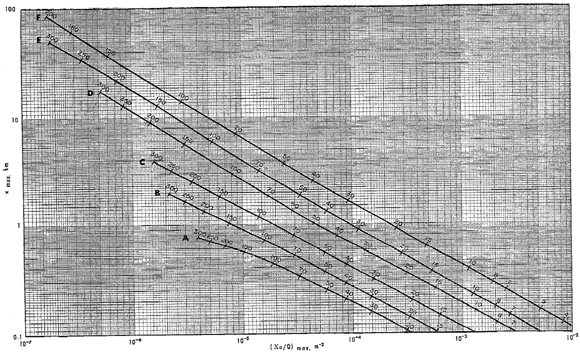

Figure 3-9. Distance of maximum concentration and maximum xu/Q as a function of stability (curves) and effective height (meters) of emission (numbers).

GENERAL CONSIDERATIONS

In most problems one must estimate the effective stack height, H, at which the plume becomes essentially level. Rarely will this height correspond to the physical height of the stack, h. If the plume is caught in the turbulent wake of the stack or of buildings in the vicinity of the stack, the effluent will be mixed rapidly downward toward the ground (aerodynamic downwash). If the plume is emitted free of these turbulent zones, a number of emission factors and meteorological factors influence the rise of the plume. The emission factors are: velocity of the effluent at the top of the stack, v_s; temperature of the effluent at the top of the stack, T_s; and diameter of the stack opening, d. The meteorological factors influencing plume rise are wind speed, u; temperature of the air, T_a; shear of the wind speed with height, du/dz; and atmospheric stability. No theory on plume rise takes into account all of these variables; even if such a theory were available, measurements of all of the parameters would seldom be available. Most of the equations that have been formulated for computing the effective height of emission are semi-empirical. For a recent review of equations for effective height of emission see Moses, Strom, and Carson (1964).

Moses and Strom (1961), having compared actual and calculated plume heights by means of six plume rise equations, report "There is no one formula which is outstanding in all respects." The formulas of Davidson-Bryant (1949), Holland (1953), Bosanquet-Carey-Halton (1950), and Bosanquet (1957) all give generally satisfactory results in the test situations. The experiments conducted by Moses and Strom involved plume rise from a stack of less than 0.5 meter diameter, stack gas exit velocities less than 15 m sec^{-1}, and effluent temperature not more than 35°C higher than that of the ambient air.

The equation of Holland was developed with experimental data from larger sources than those of Moses and Strom (stack diameters from 1.7 to 4.3 meters and stack temperatures from 82 to 204°C); Holland's equation is used in the solution of the problems given in this workbook. This equation frequently underestimates the effective height of emission; therefore its use often provides a slight "safety" factor.

Holland's equation is:

$$\Delta H = \frac{v_s d}{u}\left(1.5 + 2.68 \times 10^{-3}\, p\, \frac{T_s - T_a}{T_s}\, d\right) \quad (4.1)$$

where:

ΔH = the rise of the plume above the stack, m

v_s = stack gas exit velocity, m sec^{-1}

d = the inside stack diameter, m

u = wind speed, m sec^{-1}

p = atmospheric pressure, mb

T_s = stack gas temperature, °K

T_a = air temperature, °K

and 2.68×10^{-3} is a constant having units of mb^{-1} m^{-1}.

Holland (1953) suggests that a value between 1.1 and 1.2 times the ΔH from the equation should be used for unstable conditions; a value between 0.8 and 0.9 times the ΔH from the equation should be used for stable conditions.

Since the plume rise from a stack occurs over some distance downwind, Eq. (4.1) should not be applied within the first few hundred meters of the stack.

EFFECTIVE HEIGHT OF EMISSION AND MAXIMUM CONCENTRATION

If the effective heights of emission were the same under all atmospheric conditions, the highest ground-level concentrations from a given source would occur with the lightest winds. Generally, however, emission conditions are such that the effective stack height is an inverse function of wind speed as indicated in Eq. (4.1). The maximum ground-level concentration occurs at some intermediate wind speed, at which a balance is reached between the dilution due to wind speed and the effect of height of emission. This critical wind speed will vary with stability. In order to determine the critical wind speed, the effective stack height as a function of wind speed should first be determined. The maximum concentration for each wind speed and stability can then be calculated from Figure 3-9 as a function of effective height of emission and stability. When the maximum concentration as a function of wind speed is plotted on log-log graph paper, curves can be drawn for each stability class; the critical wind speed corresponds to the point of highest maximum concentration on the curve (see problem 14).

ESTIMATES OF REQUIRED STACK HEIGHTS

Estimates of the stack height required to produce concentrations below a given value may be made through the use of Figure 3-9 by obtaining solutions for various wind speeds. Use of this figure considers maximum concentrations at any distance from the source.

In some situations high concentrations upon the property of the emitter are of little concern, but maximum concentrations beyond the property line are of the utmost importance. For first approximations it can be assumed that the maximum concentration occurs where $\sqrt{2}\,\sigma_z = H$ and that at this distance the σ's are related to the maximum concentration by:

$$\sigma_y\,\sigma_z \cong \frac{Q}{\pi\, u\, e\, \chi_{max}} \cong \frac{0.117\,Q}{u\,\chi_{max}} \qquad (4.2)$$

Knowing the source strength, Q, and the concentration not to be exceeded χ_{max}, one can determine the necessary $\sigma_y\,\sigma_z$ for a given wind speed. Figure 4-1 shows $\sigma_y\,\sigma_z$ as a function of distance for the various stability classes. The value of $\sigma_y\,\sigma_z$ and a design distance, x_d (the distance beyond which χ is less than some pre-determined value), will determine a point on this graph yielding a stability class or point between classes. The σ_z for this stability (or point between stabilities) can then be determined from Figure 3-3. The required effective stack height for this wind speed can then be approximated by $H = \sqrt{2}\,\sigma_z$ (see problem 15). Since Eq. (4.2) is an approximation, the resulting height should be used with Eq. (3.3) to ensure that the maximum concentration is sufficiently low. If enough is known about the proposed source to allow use of an equation for effective height of emission, the relation between ΔH and u can be determined. The physical stack height required at the wind speed for which H was determined is $H - \Delta H$. The same procedure, starting with the determination of $\sigma_y\,\sigma_z$, must be used with other wind speeds to determine the maximum required physical stack height (see problem 16).

EFFECT OF EVAPORATIVE COOLING

When effluent gases are washed to absorb certain constituents prior to emission, the gases are cooled and become saturated with water vapor. Upon release of the gases from the absorption tower, further cooling due to contact with cold surfaces of ductwork or stack is likely. This cooling causes condensation of water droplets in the gas stream. Upon release of the gases from the stack, the water droplets evaporate, withdrawing the latent heat of vaporization from the air and cooling the plume. The resulting negative buoyancy reduces the effective stack height (Scorer, 1959).

EFFECT OF AERODYNAMIC DOWNWASH

The influence of mechanical turbulence around a building or stack can significantly alter the effective stack height. This is especially true with high winds, when the beneficial effect of high stack-gas velocity is at a minimum and the plume is emitted nearly horizontally. The region of disturbed flow surrounds an isolated building, generally to at least twice its height and extends downwind 5 to 10 times its height. Building the stack 2.5 times the height of the highest building adjacent to the stack usually overcomes the effects of building turbulence (Hawkins and Nonhebel, 1955). Ensuring that the exit velocity of the stack gas is more than 1.5 times the wind speed will usually prevent downwash in the wake of the stack. Most of the knowledge about the turbulent wakes around stacks and buildings has been gained through wind tunnel studies (Sherlock and Lesher, 1954; Strom, 1955-1956; Strom, et al, 1957; and Halitsky, 1962). By use of models of building shapes and stacks, one may determine the wind speeds required to cause downwash for various wind directions. With a wind tunnel the meteorological variables most easily accounted for are wind speed and wind direction (by rotation of the model within the tunnel). The emission factors that may be considered are the size and shape of the plant building; the shape, height, and diameter of the stack; the amount of emission; and the stack-gas velocity.

Through wind tunnel studies, the critical wind speeds that will cause downwash from various directions can be determined for a given set of plant factors. The average number of hours of downwash per year can then be calculated by determining the frequency of wind speeds greater than the critical speeds for each direction (Sherlock and Lesher, 1954) if climatological data representative of the site are available.

Maximum downwash about a rectangular structure occurs when the direction of the wind is at an angle of 45 degrees from the major axis of the structure; minimum downwash occurs with wind flow parallel to the major axis of the structure (Sherlock and Lesher, 1954).

Halitsky (1961, 1963) has shown that the effluent from flush openings on flat roofs frequently flows in a direction opposite to that of the free atmospheric wind, owing to counter-flow along the roof in the turbulent wake above the building. In addition to the effect of aerodynamic downwash upon the release of air pollutants from stacks and buildings, one must also consider the effects of aerodynamic downwash when exposing meteorological instruments near or upon buildings.

Where the pollution is emitted from a vent or opening on a building and is immediately influenced by the turbulent wake of the building, the pollution is rapidly distributed within this turbulent wake. To account for mixing in the turbulent wake, one may assume binormal distributions of concentrations at the source, with horizontal and vertical standard deviations of σ_{y0} and σ_{z0}. The standard deviations are related to the width and height of the building, for example, letting 4.3 σ_{y0} equal the width of the building and 2.15 σ_{z0} equal

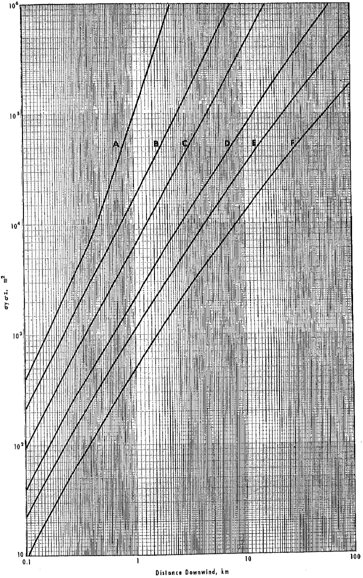

Figure 4-1. The product of $\sigma_y \sigma_z$ as a function of downwind distance from the source.

the height. Values other than 4.3 and 2.15 can be used. When these values are used 97% of the distribution is included within these limits. Virtual distances x_y and x_z can be found such that at x_y, $\sigma_y = \sigma_{y0}$ and at x_z, $\sigma_z = \sigma_{z0}$. These x's will differ with stability. Equations applicable to point sources can then be used, determining σ_y as a function of $x + x_y$ and σ_z as a function of $x + x_z$.

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CONCENTRATIONS IN AN INVERSION BREAK-UP FUMIGATION

A surface-based inversion may be eliminated by the upward transfer of sensible heat from the ground surface when that surface is warmer than the overlying air. This situation occurs when the ground is being warmed by solar radiation or when air flows from a cold to a relatively warm surface. In either situation pollutants previously emitted above the surface into the stable layer will be mixed vertically when they are reached by the thermal eddies, and ground-level concentrations can increase. This process, called "fumigation" was described by Hewson and Gill (1944) and Hewson (1945). Equations for estimating concentrations with these conditions have been given by Holland (1953), Hewson (1955), Gifford (1960a), Bierly and Hewson (1962), and Pooler (1965).

To estimate ground-level concentrations under inversion break-up fumigations, one assumes that the plume was initially emitted into a stable layer. Therefore, σ_y and σ_z characteristic of stable conditions must be selected for the particular distance of concern. An equation for the ground-level concentration when the inversion has been eliminated to a height h_i is:

$$\chi_F\,(x,y,0;H) = \frac{Q\left[\int_{-\infty}^{p} \frac{1}{\sqrt{2\pi}} \exp\,(-0.5\,p^2)\,dp\right]}{\sqrt{2\pi}\,\sigma_{yF}\,u\,h_i} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_{yF}}\right)^2\right] \tag{5.1}$$

where $p = \frac{h_i - H}{\sigma_z}$

and σ_{yF} is discussed below.

Values for the integral in brackets can be found in most statistical tables. For example, see pages 273-276, Burington (1953). This factor accounts for the portion of the plume that is mixed downward. If the inversion is eliminated up to the effective stack height, half of the plume is presumed to be mixed downward, the other half remaining in the stable air above. Eq. (5.1) can be approximated when the fumigation concentration is near its maximum by:

$$\chi_F\,(x,y,0;H) = \frac{Q}{\sqrt{2\pi}\,u\,\sigma_{yF}\,h_i} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_{yF}}\right)^2\right] \tag{5.2}$$

$$h_i = H + 2\,\sigma_z = h + \Delta H + 2\,\sigma_z \tag{5.3}$$

A difficulty is encountered in estimating a reasonable value for the horizontal dispersion since in mixing the stable plume through a vertical depth some additional horizontal spreading occurs (see problem 12). If this spreading is ignored and the σ_y for stable conditions used, the probable result would be estimated concentrations higher than actual concentrations. Or, using an approximation suggested by Bierly and Hewson (1962) that the edge of the plume spreads outward with an angle of 15°, the σ_{yF} for the inversion break-up fumigation equals the σ_y for stable conditions plus one-eighth the effective height of emission. The origin of this concept can be seen in Figure 5-1 and the following equation, where the edge of the plume is the point at which the concentration falls to 1/10 that at the centerline (at a distance of 2.15 σ_y from the plume center).

$$\sigma_{yF} = \frac{2.15\,\sigma_y\,(\text{stable}) + H\tan 15°}{2.15}$$

$$= \sigma_y\,(\text{stable}) + H/8 \tag{5.4}$$

A Gaussian distribution in the horizontal is assumed.

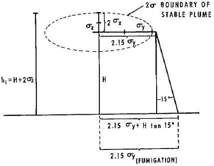

Figure 5-1. Diagram showing assumed height, h_i and σ_y during fumigation, for use in equation (5.2).

Eq. (5.4) should not be applied near the stack, for if the inversion has been eliminated to a height sufficient to include the entire plume, the emission is taking place under unstable not stable conditions. Therefore, the nearest downwind distance to be considered for an estimate of fumigation concentrations must be great enough, based on the time required to eliminate the inversion, that this portion of the plume was initially emitted into stable air. This distance is $x = ut_m$, where u is the mean wind in the stable layer and t_m is the time required to eliminate the inversion from h, the physical height of the stack to h_i (Eq. 5.3).

t_m is dependent upon both the strength of the inversion and the rate of heating at the surface. Pooler (1965) has derived an expression for estimating this time:

$$t_m = \frac{\rho_a c_p}{R} \frac{\delta \Theta}{\delta z} (h_i - h) \left(\frac{h + h_i}{2}\right) \quad (5.5)$$

where t_m = time required for the mixing layer to develop from the top of the stack to the top of the plume, sec

ρ_a = ambient air density, g m^{-3}

c_p = specific heat of air at constant pressure, cal g^{-1} °K^{-1}

R = net rate of sensible heating of an air column by solar radiation, cal m^{-2} sec^{-1}

$\frac{\delta \Theta}{\delta z}$ = vertical potential temperature gradient, °K $m^{-1} \sim \frac{\delta T}{\delta z} + \Gamma$ (the adiabatic lapse rate)

h_i = height of base of the inversion sufficient to be above the plume, m

h = physical height of the stack, m

Note that h_i —h is the thickness of the layer to be heated and $\left(\frac{h + h_i}{2}\right)$ is the average height of the layer. Although R depends on season, and cloud cover and varies continuously with time, Pooler has used a value of 67 cal m^{-2} sec^{-1} as an average for fumigation.

Hewson (1945) also suggested a method of estimating the time required to eliminate an inversion to a height z by use of an equation of Taylor's (1915, p. 8):

$$t = \frac{z^2}{4K} \quad (5.6)$$

where: t = time required to eliminate the inversion to height z, sec

z = height to which the inversion has been eliminated, m

K = eddy diffusivity for heat, m^2 sec^{-1}

Rewriting to compare with Eq. (5.5),

$$t_m = \frac{h_i^2 - h^2}{4K} \quad (5.7)$$

Hewson (1945) has suggested a value of 3 m^2 sec^{-1} for K.

PLUME TRAPPING

Plume trapping occurs when the plume is trapped between the ground surface and a stable layer aloft. Bierly and Hewson (1962) have suggested the use of an equation that accounts for the multiple eddy reflections from both the ground and the stable layer:

$$\chi(x,0,z;H) = \frac{Q}{2\pi u \sigma_y \sigma_z} \Bigg\{ \exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right] + \sum_{N=1}^{N=J} \Bigg[\exp - \frac{1}{2}\left(\frac{z-H-2NL}{\sigma_z}\right)^2 + \exp - \frac{1}{2}\left(\frac{z+H-2NL}{\sigma_z}\right)^2 + \exp - \frac{1}{2}\left(\frac{z-H+2NL}{\sigma_z}\right)^2 + \exp - \frac{1}{2}\left(\frac{z+H+2NL}{\sigma_z}\right)^2 \Bigg] \Bigg\} \quad (5.8)$$

where L is the height of the stable layer and J = 3 or 4 is sufficient to include the important reflections. A good approximation of this lengthy equation can be made by assuming no effect of the stable layer until $\sigma_z = 0.47$ L (see Chapter 3). It is assumed that at this distance, x_L, the stable layer begins to affect the vertical distribution so that at the downwind distance, 2 x_L, uniform vertical mixing has taken place and the following equation can be used:

$$\chi(x,y,z;H) = \frac{Q}{\sqrt{2\pi}\,\sigma_y L u} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \quad (5.9)$$

For distances between x_L and 2 x_L the best approximation to the ground-level centerline concentration is that read from a straight line drawn between the concentrations for points x_L and 2 x_L on a log-log plot of ground-level centerline concentration as a function of distance.

CONCENTRATIONS AT GROUND LEVEL COMPARED TO CONCENTRATIONS AT THE LEVEL OF EFFECTIVE STACK HEIGHT FROM ELEVATED CONTINUOUS SOURCES

There are several interesting relationships between ground-level concentrations and concentrations at the level of the plume centerline. One of these is at the distance of maximum concentration at the ground. As a rough approximation the maximum ground-level concentration occurs at the distance where $\sigma_z = \frac{1}{\sqrt{2}} H$. This approximation is much better for unstable conditions than for stable conditions. With this approximation, the ratio of concentration at plume centerline to that at the ground is:

$$\frac{\chi(x,0,H)}{\chi(x,0,0)} = \frac{\frac{1}{2}\left[1.0 + \exp - \frac{1}{2}\left(\frac{2H}{\sigma_z}\right)^2\right]}{\exp - \frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2}$$

$$= \frac{\frac{1}{2}\left[1.0 + \exp - 0.5\,(2\sqrt{2})^2\right]}{\exp - 0.5\,(\sqrt{2})^2}$$

$$= \frac{\frac{1}{2}(1.0 + 0.0182)}{0.368}$$

$$= 1.38$$

This calculation indicates that at the distance of maximum ground-level concentration the concentration at plume centerline is greater by about one-third.

It is also of interest to determine the relationship between σ_z and H such that the concentration at ground-level at a given distance from the source is the same as the concentration at plume level. This condition should occur where:

$$\exp - \frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2 = \frac{1}{2}\left[1.0 + \exp - \frac{1}{2}\left(\frac{2H}{\sigma_z}\right)^2\right]$$

The value $H/\sigma_z = 1.10$ satisfies this expression, which can be written as $\sigma_z = 0.91\ H$ (see problem 10).

TOTAL DOSAGE FROM A FINITE RELEASE

The total dosage, which is the integration of concentration over the time of passage of a plume or puff, can be obtained from:

$$D_T(x,y,0;H) = \frac{Q_T}{\pi\,\sigma_y\,\sigma_z\,u} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \qquad (5.10)$$

where D_T = total dosage, g sec m^{-3}
and Q_T = total release, g

The σ's should be representative of the time period over which the release takes place, and care should be taken to consider the x-axis along the trajectory or path of the plume or puff travel. Large errors can easily occur if the path is not known accurately. The estimate of this path is usually increasingly difficult with shorter release times. D_T can also be given in curie sec m^{-3} if Q_T is in curies.

CROSSWIND-INTEGRATED CONCENTRATION

The ground-level crosswind-integrated concentration is often of interest. For a continuous elevated source this concentration is determined from Eq. (3.2) integrated with respect to y from $-\infty$ to $+\infty$ (Gifford 1960a) giving:

$$\chi_{CWI} = \frac{2\,Q}{\sqrt{2\pi}\,\sigma_z\,u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \qquad (5.11)$$

In diffusion experiments the ground-level crosswind-integrated concentration is often determined at particular downwind distances from a crosswind line or arc of sampling measurements made at this distance. When the source strength, Q, and average wind speed, u, are known, σ_z can be estimated indirectly even though no measurements were made in the vertical. If any of the tracer is lost through reaction or deposition, the resulting σ_z from such estimates will not represent the vertical dispersion (see problem 18).

ESTIMATION OF CONCENTRATIONS FOR SAMPLING TIMES LONGER THAN A FEW MINUTES

Concentrations directly downwind from a source decrease with sampling time mainly because of a larger σ_y due to increased meander of wind direction. Stewart, Gale, and Crooks (1958) reported that this decrease in concentration follows a one-fifth power law with the sampling time for sampling periods from about 3 minutes to about half an hour. Cramer (1959) indicates that this same power law applies for sampling times from 3 seconds to 10 minutes. Both of these studies were based on observations taken near the height of release. Gifford (1960b) indicates that ratios of peak to mean concentrations are much higher than those given by the above power law where observations of concentrations are made at heights considerably different from the height of release or considerably removed from the plume axis. He also indicates that for increasing distances from an elevated source, the ratios of peak to average concentrations observed at ground level approach unity. Singer (1961) and Singer, et al. (1963) show that ratios of peak to mean concentrations depend also on the stability of the atmosphere and the type of terrain that the plume is passing over. Nonhebel (1960) reports that Meade deduced a relation between calculated concentrations at ground level and the sampling time from "a study of published data on lateral and vertical diffusion coefficients in steady winds." These relations are shown in Table 5-1.

Table 5-1 VARIATION OF CALCULATEO CONCENTRATION WITH SAMPLING TIME

Sampling Time	Ratio of Calculated Concentration to 3-minute Concentration
3 minutes	1.00
15 minutes	0.82
1 hour	0.61
3 hours	0.51
24 hours	0.36

This table indicates a power relation with time: $\chi \propto t^{-0.17}$. Note that these estimates were based upon published dispersion coefficients rather than upon sampling results. Information in the references cited indicates that effects of sampling time are exceedingly complex. If it is necessary to estimate concentrations from a single source for the time intervals greater than a few minutes, the best estimate apparently can be obtained from:

$$\chi_s = \chi_k \left(\frac{t_k}{t_s}\right)^p \qquad (5.12)$$

vhere χ_s is the desired concentration estimate for he sampling time, t_s; χ_k is the concentration estimate for the shorter sampling time, t_k, (probably ıbout 10 minutes); and p should be between 0.17 ınd 0.2. Eq. (5.12) probably would be applied nost appropriately to sampling times less than 2 ıours (see problem 19).

ESTIMATION OF SEASONAL OR ANNUAL AVERAGE CONCENTRATIONS AT A RECEPTOR FROM A SINGLE POLLUTANT SOURCE

For a source that emits at a constant rate from ıour to hour and day to day, estimates of seasonal ɔr annual average concentrations can be made for any distance in any direction if stability wind "rose" data are available for the period under study. A wind rose gives the frequency of occurrence for each wind direction (usually to 16 points) and wind speed class (9 classes in standard Weather Bureau use) for the period under consideration (from 1 month to 10 years). A stability wind rose gives the same type of information for each stability class.

If the wind directions are taken to 16 points and it is assumed that the wind directions within each sector are distributed randomly over a period of a month or a season, it can further be assumed that the effluent is uniformly distributed in the horizontal within the sector (Holland, 1953, p. 540). The appropriate equation for average concentration is then either:

$$\bar{\chi} = \frac{2\,Q}{\sqrt{2\pi}\,\sigma_z\, u \left(\frac{2\pi x}{16}\right)} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]$$

$$= \frac{2.03Q}{\sigma_z\, u x} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \qquad (5.13)$$

or

$$\bar{\chi} = \frac{Q}{L\,u\left(\frac{2\pi x}{16}\right)} = \frac{2.55\,Q}{L\,u\,x} \qquad (5.14)$$

depending upon whether a stable layer aloft is affecting the distribution.

The estimation of χ for a particular direction and downwind distance can be accomplished by choosing a representative wind speed for each speed class and solving the appropriate equation (5.13 or 5.14) for all wind speed classes and stabilities. Note that a SSW wind affects a receptor to the NNE of a source. One obtains the average concentration for a given direction and distance by summing all the concentrations and weighting each one according to its frequency for the particular stability and wind speed class. If desired, a different effective height of emission can be used for various wind speeds. The average concentration can be expressed by:

$$\chi(x,\theta) = \sum_{S}\sum_{N}\left\{\frac{2\,Q\,f(\theta,S,N)}{\sqrt{2\pi}\,\sigma_{zS}\,u_N\left(\frac{2\pi x}{16}\right)} \exp\left[-\frac{1}{2}\left(\frac{H_u}{\sigma_{zS}}\right)^2\right]\right\} \qquad (5.15)$$

where $f(\theta, S, N)$ is the frequency during the period of interest that the wind is from the direction θ, for the stability condition, S, and wind speed class N.

σ_{zS} is the vertical dispersion parameter evaluated at the distance x for the stability condition S.

u_N is the representative wind speed for class N.

H_u is the effective height of release for the wind speed u_N.

Where stability wind rose information cannot be obtained, a first-order approximation may be made of seasonal or annual average concentrations by using the appropriate wind rose in the same manner, and assuming the neutral stability class, D, only.

METEOROLOGICAL CONDITIONS ASSOCIATED WITH MAXIMUM GROUND-LEVEL CONCENTRATIONS

1. For ground-level sources maximum concentrations occur with stable conditions.

2. For elevated sources maximum *"instantaneous"* concentrations occur with unstable conditions when portions of the plume that have undergone little dispersion are brought to the ground. These occur close to the point of emission (on the order of 1 to 3 stack heights). These concentrations are usually of little general interest because of their very short duration; they *cannot* be estimated from the material presented in this workbook.

3. For elevated sources maximum concentrations for time periods of a few minutes occur with unstable conditions; although the concentrations fluctuate considerably under these conditions, the concentrations averaged over a few minutes are still high compared to those found under other conditions. The distance of this maximum concentration occurs near the stack (from 1 to 5 stack heights downwind) and the concentration drops off rapidly downwind with increasing distance.

4. For elevated sources maximum concentrations for time periods of about half an hour can occur with fumigation conditions when an unstable layer increases vertically to mix downward a plume previously discharged within a stable layer. With small ΔH, the fumigation can occur close to the source but will be of relatively short duration. For large ΔH, the fumigation will occur some distance from the stack (perhaps 30 to 40 km), but can persist for a longer time interval. Concentrations considerably lower than those associated with fumigations, but of significance can occur with neutral or unstable conditions when the dispersion upward is severely limited by the existence of a more stable layer above the plume, for example, an inversion.

5. Under stable conditions the maximum concentrations at ground-level from elevated sources are less than those occurring under unstable conditions and occur at greater distances from the source. However, the difference between maximum ground-level concentrations for stable and unstable conditions is only a factor of 2 for effective heights of 25 meters and a factor of 5 for H of 75 m. Because the maximum occurs at greater distances, concentrations that are below the maximum but still significant can occur over large areas. This becomes increasingly significant if emissions are coming from more than one source.

CONCENTRATIONS AT A RECEPTOR POINT FROM SEVERAL SOURCES

Sometimes, especially for multiple sources, it is convenient to consider the receptor as being at the origin of the diffusion coordinate system. The source-receptor geometry can then be worked out merely by drawing or visualizing an x-axis oriented upwind from the receptor and determining the crosswind distances of each source in relation to this x-axis. As pointed out by Gifford (1959), the concentration at (0, 0, 0) from a source at (x, y, H) on a coordinate system with the x-axis oriented upwind is the same as the concentration at (x, y, 0) from a source at (0, 0, H) on a coordniate system with the x-axis downwind (Figure 5-2). The total concentration is then given by summing the individual contributions from each source (see problem 20).

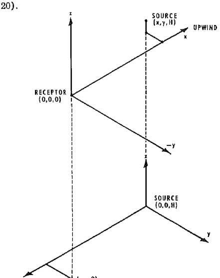

Figure 5-2. Comparison of source-oriented and receptor-oriented coordinate systems.

It is often difficult to determine the atmospheric conditions of wind direction, wind speed, and stability that will result in the maximum combined concentrations from two or more sources; drawing isopleths of concentration for various wind speeds and stabilities and orienting these according to wind direction is one approach.

AREA SOURCES

In dealing with diffusion of air pollutants in areas having large numbers of sources, e.g., as in urban areas, there may be too many sources of most atmospheric contaminants to consider each source individually. Often an approximation can be made by combining all of the emissions in a given area and treating this area as a source having an initial horizontal standard deviation, σ_{yo}. A virtual distance, x_y, can then be found that will give this standard deviation. This is just the distance that will yield the appropriate value for σ_y from Figure 3-2. Values of x_y will vary with stability. Then equations for point sources may be used, determining σ_y as a function of $x + x_y$, a slight variation of the suggestion by Holland (1953). This procedure treats the area source as a cross-wind line source with a normal distribution, a fairly good approximation for the distribution across an area source. The initial standard deviation for a square area source can be approximated by $\sigma_{yo} \cong s/4.3$, where s is the length of a side of the area (see problem 22).

If the emissions within an area are from varying effective stack heights, the variation may be approximated by using a σ_{zo}. Thus H would be the *mean* effective height of release and σ_{zo} the standard deviation of the initial vertical distribution of sources. A virtual distance, x_z, can be found, and point source equations used for estimating concentrations, determining σ_z as a function of $x + x_z$.

TOPOGRAPHY

Under conditions of irregular topography the direct application of a standard dispersion equation is often invalid. In some situations the best one may be able to do without the benefit of *in situ* experiments is to estimate the upper limit of the concentrations likely to occur.

For example, to calculate concentrations on a hillside downwind from and facing the source and at about the effective source height, the equation for concentrations at ground-level from a ground-level source (Eq. 3.4) will yield the highest expected concentrations. This would closely approximate the situation under stable conditions, when the pollutant plume would be most likely to encounter the hillside. Under unstable conditions the flow is more likely to rise over the hill (see problem 21).

With downslope flow when the receptor is at a lower elevation than the source, a likely assumption is that the flow parallels the slope; i.e., no allowance is made for the difference between ground-level elevations at the source and at the receptor.

Where a steep ridge or bluff restricts the horizontal dispersion, the flow is likely to be parallel to such a bluff. An assumption of complete reflection at the bluff, similar to eddy reflection at the ground from an elevated source, is in order. This may be accomplished by using:

$$\chi(x,y,0;H) = \frac{Q}{\pi \sigma_y \sigma_z u} \left\{ \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{2B-y}{\sigma_y}\right)^2\right]\right\} \left\{ \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]\right\} \quad (5.16)$$

B is the distance from the x-axis to the restricting bluff, and the positive y axis is defined to be in the direction of the bluff.

The restriction of horizontal dispersion by valley sides is somewhat analogous to restriction of the vertical dispersion by a stable layer aloft. When the σ_y becomes great enough, the concentrations can be assumed to be uniform across the width of the valley and the concentration calculated according to the following equation, where in this case Y is the width of the valley.

$$\chi = \frac{2Q}{\sqrt{2\pi}\,\sigma_z Y u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \quad (5.17)$$

LINE SOURCES

Concentrations downwind of a continuously emitting infinite line source, when the wind direction is normal to the line, can be expressed by rewriting equation (12) p. 154 of Sutton (1932):

$$\chi(x,y,0;H) = \frac{2q}{\sqrt{2\pi}\,\sigma_z u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \quad (5.18)$$

Here q is the source strength per unit distance, for example, g sec^{-1} m^{-1}. Note that the horizontal dispersion parameter, σ_y, does not appear in this equation, since it is assumed that lateral dispersion from one segment of the line is compensated by dispersion in the opposite direction from adjacent segments. Also y does not appear, since concentration at a given x is the same for any value of y (see problem 23).

Concentrations from infinite line sources when the wind is not perpendicular to the line can be approximated. If the angle between the wind direction and line source is ϕ, the equation for concentration downwind of the line source is:

$$\chi(x,y,0;H) = \frac{2q}{\sin\phi \sqrt{2\pi}\,\sigma_z u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \quad (5.19)$$

This equation should not be used where ϕ is less than 45°.

When estimating concentrations from finite line sources, one must account for "edge effects" caused by the end of the line source. These effects will of course extend to greater cross-wind distances as the distance from the source increases. For concentrations from a finite line source oriented cross-wind, define the x-axis in the direction of the mean wind and passing through the receptor of interest. The limits of the line source can be defined as extending from y_1 to y_2 where y_1 is less than y_2. The equation for concentration (from Sutton's (1932) equation (11), p. 154), is:

$$\chi\,(x,0,0;H) = \frac{2\,q}{\sqrt{2\pi}\,\sigma_z\,u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \int_{p_1}^{p_2} \frac{1}{\sqrt{2\pi}} \exp\,(-0.5\,p^2)\,dp \qquad (5.20)$$

$$\text{where } p_1 = \frac{y_1}{\sigma_y},\; p_2 = \frac{y_2}{\sigma_y}$$

The value of the integral can be determined from tabulations given in most statistical tables (for example, see Burrington (1953), pp. 273-276; also see problem 24).

INSTANTANEOUS SOURCES

Thus far we have considered only sources that were emitting continuously or for time periods equal to or greater than the travel times from the source to the point of interest. Cases of instantaneous release, as from an explosion, or short-term releases on the order of seconds, are often of practical concern. To determine concentrations at any position downwind, one must consider the time interval after the time of release and diffusion in the downwind direction as well as lateral and vertical diffusion. Of considerable importance, but very difficult, is the determination of the path or trajectory of the "puff." This is most important if concentrations are to be determined at specific points. Determining the trajectory is of less importance if knowledge of the magnitude of the concentrations for particular downwind distances or travel times is required without the need to know exactly at what points these concentrations occur. Rewriting Sutton's (1932) equation (13), p. 155, results in an equation that may be used for estimates of concentration downwind from a release from height, H:

$$\chi\,(x,y,0;H) = \frac{2\,Q_T}{(2\pi)^{3/2}\,\sigma_x\,\sigma_y\,\sigma_z} \exp\left[-\frac{1}{2}\left(\frac{x-ut}{\sigma_x}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \qquad (5.21)$$

(The numerical value of $(2\pi)^{3/2}$ is 15.75.)

The symbols have the usual meaning, with the important exceptions that Q_T represents the *total mass* of the release and the σ's are *not* those evaluated with respect to the dispersion of a continuous source at a fixed point in space.

In Eq. (5.21) the σ's refer to dispersion statistics following the motion of the expanding puff. The σ_x is the standard deviation of the concentration distribution in the puff in the downwind direction, and t is the time after release. Note that there is no dilution in the downwind direction by wind speed. The speed of the wind mainly serves to give the downwind position of the center of the puff, as shown by examination of the exponential involving σ_x. Wind speed may influence the dispersion indirectly because the dispersion parameters σ_x, σ_y, and σ_z may be functions of wind speed. The σ_y's and σ_z's for an instantaneous source are less than those for a few minutes given in Figure 3-2 and 3-3. Slade (1965) has suggested values for a σ_y and σ_z for quasi-instantaneous sources. These are given in Table 5-2. The problem remains to make best estimates of σ_x. Much less is known of diffusion in the downwind direction than is known of lateral and vertical dispersion. In general one should expect the σ_x value to be about the same as σ_y. Initial dimensions of the puff, i.e., from an explosion, may be approximated by finding a virtual distance to give the appropriate initial standard deviation for each direction. Then σ_y will be determined as a function of $x + x_y$, σ_z as a function of $x + x_z$, and σ_x as a function of $x + x_x$.

Table 5-2 ESTIMATION OF DISPERSION PARAMETERS FOR QUASI-INSTANTANEOUS SOURCES (FROM SLADE, 1965)

	$x = 100$ m		$x = 4$ km	
	σ_y	σ_z	σ_y	σ_z
Unstable	10	15	300	220
Neutral	4	3.8	120	50
Very Stable	1.3	0.75	35	7

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Chapter 6 — RELATION TO OTHER DIFFUSION EQUATIONS

Most other widely used diffusion equations are variant forms of the ones presented here. With respect to ground-level concentrations from an elevated source (Eq. 3.2):

$$\chi\,(x,y,0;H) = \frac{Q}{\pi\,\sigma_y\,\sigma_z\,u}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \qquad (3.2)$$

Other well-known equations can be compared:

Bosanquet and Pearson (1936):

$$\chi\,(x,y,0;H) = \frac{Q}{\sqrt{2\pi}\,pq\,x^2\,u}\exp\left[-\frac{1}{2}\left(\frac{y}{qx}\right)^2\right]\exp\left[-\frac{H}{px}\right] \qquad (6.1)$$

where p and q are dimensionless diffusion coefficients.

Sutton (1947):

$$\chi\,(x,y,0;H) = \frac{2\,Q}{\pi\,C_y\,C_z\,x^{2-n}\,u}\exp\left[-\frac{1}{x^{2-n}}\left(\frac{y^2}{C_y^2}+\frac{H^2}{C_z^2}\right)\right] \qquad (6.2)$$

where n is a dimensionless constant and C_y and C_z are diffusion coefficients in $m^{n/2}$.

Calder (1952):

$$\chi\,(x,y,0;H) = \frac{Q\,u}{2\,k^2\,a\,v_x^2\,x^2}\exp\left[-\frac{u}{k\,v_x\,x}\left(\frac{y}{a}+H\right)\right] \qquad (6.3)$$

where $a = \frac{v'}{w'}$, the ratio of horizontal eddy velocity to vertical eddy velocity, k is von Karman's constant approximately equal to 0.4, and $v_x = \frac{k\,u}{\ln\left(\frac{H}{z_0}\right)}$ where z_0 is a roughness parameter, m.

NOTE: Calder wrote the equation for the concentration at (x, y, z) from a ground-level source. For Eq. (6.3) it is assumed that the concentration at ground level from an elevated source is the same as the concentraton at an elevated point from a ground-level source.

Table 6-1 lists the expressions used in these equations that are equivalent to σ_y and σ_z (continuous source) in this paper.

Table 6-1 EXPRESSIONS EQUIVALENT TO σ_y AND σ_z IN VARIOUS DIFFUSION EQUATIONS.

Equation	σ_y	σ_z
Bosanquet and Pearson	$q\,x$	$\sqrt{2}\,p\,x$
Sutton	$\frac{1}{\sqrt{2}}\,C_y\,x^{\frac{2-n}{2}}$	$\frac{1}{\sqrt{2}}\,C_z\,x^{\frac{2-n}{2}}$
Calder	$\frac{\sqrt{2}\,a\,k\,v_x\,x}{u}$	$\frac{\sqrt{2}\,k\,v_x\,x}{u}$

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The following 26 example problems and their solutions illustrate the application of most of the techniques and equations presented in this workbook.

PROBLEM 1: It is estimated that a burning dump emits 3 g sec^{-1} of oxides of nitrogen. What is the concentration of oxides of nitrogen, averaged over approximately 10 minutes, from this source directly downwind at a distance of 3 km on an overcast night with wind speed of 7 m sec^{-1}? Assume this dump to be a point ground-level source with no effective rise.

SOLUTION: Overcast conditions with a wind speed of 7 m sec^{-1} indicate that stability class D is most applicable (Statement, bottom of Table 3-1). For x = 3 km and stability D, $\sigma_y = 190$ m from Figure 3-2 and $\sigma_z = 65$ m from Figure 3-3. Eq. (3.4) for estimation of concentrations directly downwind (y = 0) from a ground-level source is applicable:

$$\chi\ (x,0,0;0) = \frac{Q}{\pi\ \sigma_y\ \sigma_z\ u} = \frac{3}{\pi\ 190\ (65)\ 7}$$

$= 1.1 \times 10^{-5}$ g m^{-3} of oxides of nitrogen.

PROBLEM 2: It is estimated that 80 g sec^{-1} of sulfur dioxide is being emitted from a petroleum refinery from an average effective height of 60 meters. At 0800 on an overcast winter morning with the surface wind 6 m sec^{-1}, what is the ground-level concentration directly downwind from the refinery at a distance of 500 meters?

SOLUTION: For overcast conditions, D class stability applies. With D stability at x = 500 m, $\sigma_y = 36$ m, $\sigma_z = 18.5$ m. Using Eq. (3.3):

$$\chi\ (x,0,0;H) = \frac{Q}{\pi\ \sigma_y\ \sigma_z\ u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]$$

$$= \frac{80}{\pi\ 36\ (18.5)\ 6} \exp\ [-0.5\ (60/18.5)^2]$$

$$= 6.37 \times 10^{-3} \exp\ [-0.5\ (3.24)^2]$$

The exponential is solved using Table A-1 (Appendix 3).

$$= 6.37 \times 10^{-3}\ (5.25 \times 10^{-3})$$

$\chi = 3.3 \times 10^{-5}$ g m^{-3} of SO_2

PROBLEM 3: Under the conditions of problem 2, what is the concentration at the same distance downwind but at a distance 50 meters from the x-axis? That is: χ (500, 50, 0; 60) = ?

SOLUTION: Using Eq. (3.2):

$$\chi\ (x,y,0;H) = \frac{Q}{\pi\ \sigma_y\ \sigma_z\ u} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]$$

All but the exponential involving y has been found in the preceding problem. Therefore:

$$\chi\ (500, 50, 0; 60) = 3.3 \times 10^{-5} \exp\ [-0.5\ (50/36)^2]$$

$= 3.3 \times 10^{-5}\ (0.381)$

$= 1.3 \times 10^{-5}$ g m^{-3} of SO_2

PROBLEM 4: A power plant burns 10 tons per hour of coal containing 3 percent sulfur; the effluent is released from a single stack. On a sunny summer afternoon the wind at 10 meters above ground is 4 m sec^{-1} from the northeast. The morning radiosonde taken at a nearby Weather Bureau station has indicated that a frontal inversion aloft will limit the vertical mixing to 1500 meters. The 1200-meter wind is from 30° at 5 m sec^{-1}. The effective height of emission is 150 meters. From Figure 3-9, what is the distance to the maximum ground-level concentration and what is the concentration at this point?

SOLUTION: To determine the source strength, the amount of sulfur burned is: 10 tons hr^{-1} x 2000 lb ton^{-1} x 0.03 sulfur = 600 lb sulfur hr^{-1}. Sulfur has a molecular weight of 32 and combines with O_2 with a molecular weight of 32; therefore for every mass unit of sulfur burned, there result two mass units of SO_2.

$$Q = \frac{64\ (\text{molecular weight of } SO_2)}{32\ (\text{molecular weight of sulfur})} \times \frac{600\ \text{lb hr}^{-1}\ (453.6\ \text{g lb}^{-1})}{3600\ \text{sec hr}^{-1}}$$

$= 151$ g sec^{-1} of SO_2

On a sunny summer afternoon the insolation should be strong. From Table 3-1, strong insolation and 4m sec^{-1} winds yield class-B stability. From Figure 3-9, the distance to the point of maximum concentration is 1 km for class-B stability and effective height of 150 meters. From Figure 3-3 at this distance $\sigma_z = 110$ m. This is much less than 0.47 L. Therefore, at this distance, the limit of mixing of 1500 meters will not affect the ground-level concentration. From Figure 3-9, the maximum $\chi u/Q$ for B stability and this effective height of 150 m is 7.5×10^{-6}.

$$\chi_{max} = \frac{\chi u}{Q_{max}}\ \frac{Q}{u} = \frac{7.5 \times 10^{-6} \times 151}{4}$$

$= 2.8 \times 10^{-4}$ g m^{-3} of SO_2

PROBLEM 5: For the power plant in problem 4, at what distance does the maximum ground-level concentration occur and what is this concentration on an overcast day with wind speed 4 m sec^{-1}?

SOLUTION: On an overcast day the stability class would be D. From Figure 3-9 for D stability and H of 150 m, the distance to the point of maximum ground-level concentration is 5.6 km, and the maximum $\chi u/Q$ is 3.0×10^{-6}.

$$\chi_{max} = \frac{3.0 \times 10^{-6} \times 151}{4}$$

$$= 1.1 \times 10^{-4} \text{ g m}^{-3}$$

PROBLEM 6: For the conditions given in problem 4, draw a graph of ground-level centerline sulfur dioxide concentration with distance from 100 meters to 100 km. Use log-log graph paper.

SOLUTION: The frontal inversion limits the mixing to L = 1500 meters. The distance at which $\sigma_z = 0.47$ L = 705 m is $x_L = 5.5$ km. At distances less than this, Eq. (3.3) is used to calculate concentrations:

$$\chi\ (x,0,0;H) = \frac{Q}{\pi\, \sigma_y\, \sigma_z\, u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]$$

At distance equal to or greater than 2 x_L, which is 11 km, Eq. (3.5) is used:

$$\chi\ (x,0,0;H) = \frac{Q}{\sqrt{2\pi}\, \sigma_y\, L\, u}$$

Solutions for the equations are given in Table 7-1. The values of concentration are plotted against distance in Figure 7-1.

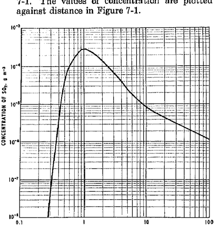

Figure 7-1. Concentration as a function of downwind distance (Problem 6).

Table 7-1 CALCULATION OF CONCENTRATIONS FOR VARIOUS DISTANCES (PROBLEM 6)

x, km	u, m sec^{-1}	σ_y, m	σ_z, m	H/σ_z	$\exp\left[-\frac{1}{2}(H/\sigma_z)^2\right]$	χ, g m^{-3}
0.3	4	52	30	5.0	3.73×10^{-6}	2.9×10^{-8}
0.5	4	83	51	2.94	1.33×10^{-2}	3.8×10^{-5}
0.8	4	129	85	1.77	0.209	2.3×10^{-4}
1.0	4	157	110	1.36	0.397	2.8×10^{-4}
2.0	4	295	230	0.65	0.810	1.4×10^{-4}
3.0	4	425	365	0.41	0.919	7.1×10^{-5}
5.5	4.5	720	705	0.21	0.978	2.1×10^{-5}
x, km	**u, m sec^{-1}**	**σ_y, m**	**L, m**			**χ, g m^{-3}**
11.0	4.5	1300	1500			6.9×10^{-6}
30	4.5	3000	1500			3.0×10^{-6}
100	4.5	8200	1500			1.1×10^{-6}

PROBLEM 7: For the conditions given in problem 4, draw a graph of ground-level concentration versus crosswind distance at a downwind distance of 1 km.

SOLUTION: From problem 4 the ground-level centerline concentration at 1 km is 2.8×10^{-4} g m^{-3}. To determine the concentrations at distances y from the x-axis, the ground-level centerline concentration must be multiplied by the factor $\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$

$\sigma_y = 157$ meters at x = 1 km. Values for this computation are given in Table 7-2.

Table 7-2 DETERMINATION OF CROSSWIND CONCENTRATIONS (PROBLEM 7)

y, m	$\frac{y}{\sigma_y}$	$\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$	χ (x,y,0)
± 100	0.64	0.815	2.3×10^{-4}
± 200	1.27	0.446	1.3×10^{-4}
± 300	1.91	0.161	4.5×10^{-5}
± 400	2.55	3.87×10^{-2}	1.1×10^{-5}
± 500	3.18	6.37×10^{-3}	1.8×10^{-6}

These concentrations are plotted in Figure 7-2.

PROBLEM 8: For the conditions given in problem 4, determine the position of the 10^{-5} g m^{-3} ground level isopleth, and determine its area.

SOLUTION: From the solution to problem 6, the graph (Figure 7-1) shows that the 10^{-5} g m^{-3} isopleth intersects the x-axis at approximately x = 350 meters and x = 8.6 kilometers.

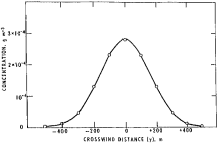

Figure 7-2. Concentration as a function of crosswind distance (Problem 7).

The values necessary to determine the isopleth half widths, y, are given in Table 7-3.

Table 7-3 DETERMINATION OF ISOPLETH WIDTHS (PROBLEM 8)

x, km	σ_y, m	χ (centerline), g m^{-3}	χ (isopleth) / χ (centerline)	y/σ_y	y, m
0.5	83	3.8 x 10^{-5}	0.263	1.64	136
0.8	129	2.3 x 10^{-4}	4.35 x 10^{-2}	2.50	323
1.0	157	2.8 x 10^{-4}	3.53 x 10^{-2}	2.59	407
2.0	295	1.4 x 10^{-4}	7.14 x 10^{-2}	2.30	679
3.0	425	7.1 x 10^{-5}	1.42 x 10^{-1}	1.98	842
4.0	540	4.0 x 10^{-5}	0.250	1.67	902
5.0	670	2.4 x 10^{-5}	0.417	1.32	884
6.0	780	1.8 x 10^{-5}	0.556	1.08	842
7.0	890	1.4 x 10^{-5}	0.714	0.82	730
8.0	980	1.1 x 10^{-5}	0.909	0.44	432

The orientation of the x-axis will be toward 225° close to the source, curving more toward 210° to 215° azimuth at greater distances because of the change of wind direction with height. The isopleth is shown in Figure 7-3.

Since the isopleth approximates an ellipse, the area may be estimated by π ab where a is the semimajor axis and b is the semiminor axis.

$$a = \frac{8600 - 350}{2} = 4125 \text{ m}$$

$$b = 902$$

$$A\ (m^2) = \pi\ (4125)\ (902)$$

$$= 11.7 \times 10^6\ m^2$$

or $A = 11.7\ km^2$

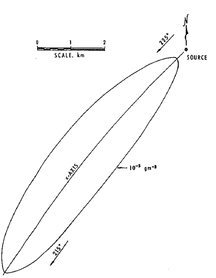

Figure 7-3. Location of the 10^{-5} g m^{-3} ground-level isopleth (Problem 8).

PROBLEM 9: For the conditions given in problem 4, determine the profile of concentration with height from ground level to z = 450 meters at x = 1 km, y = 0 meters, and draw a graph of concentration against height above ground.

SOLUTION: Eq. (3.1) is used to solve this problem. The exponential involving y is equal to 1. At x = 1 km, σ_y = 157 m, σ_z = 110 m. (From problem 4).

$$\frac{Q}{2\pi\ \sigma_y\ \sigma_z\ u} = \frac{151}{2\pi\ 157\ (110)\ 4} = 3.5 \times 10^{-5}\ g\ m^{-3}$$

Values for the estimation of $\chi(z)$ are given in Table 7-4.

PROBLEM 10: For the conditions given in problem 4, determine the distance at which the ground-level centerline concentration equals the centerline concentration at 150 meters above ground. Verify by computation of χ (x,0,0) and χ (x,0,150).

SOLUTION: The distance at which concentrations at the ground and at plume height are equal should occur where σ_z = 0.91 H (See Chapter 5). For B stability and H = 150 m, σ_z = 0.91 (150) = 136 m occurs at x = 1.2 km. At this distance σ_y = 181 m.

Table 7-4 DETERMINATION OF CONCENTRATIONS FOR VARIOUS HEIGHTS (PROBLEM 9)

a.	b.	c.	d.	e.	f.	g.
z, m	$\frac{z-H}{\sigma_z}$	$\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right]$	$\frac{z+H}{\sigma_z}$	$\exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right]$	c. + e.	$\chi(z)$, g m^{-3}
0	—1.36	0.397	1.36	0.397	0.794	2.78×10^{-4}
30	—1.09	0.552	1.64	0.261	0.813	2.85×10^{-4}
60	—0.82	0.714	1.91	0.161	0.875	3.06×10^{-4}
90	—0.55	0.860	2.18	0.0929	0.953	3.34×10^{-4}
120	—0.27	0.964	2.45	0.0497	1.014	3.55×10^{-4}
150	0.0	1.0	2.73	0.0241	1.024	3.58×10^{-4}
180	0.27	0.964	3.00	1.11×10^{-2}	0.975	3.41×10^{-4}
210	0.55	0.860	3.27	4.77×10^{-3}	0.865	3.03×10^{-4}
240	0.82	0.714	3.54	1.90×10^{-3}	0.716	2.51×10^{-4}
270	1.09	0.552	3.82	6.78×10^{-4}	0.553	1.94×10^{-4}
300	1.36	0.397	4.09	2.33×10^{-4}	0.397	1.39×10^{-4}
330	1.64	0.261	4.36	7.45×10^{-5}	0.261	9.14×10^{-5}
360	1.91	0.161	4.64	2.11×10^{-5}	0.161	5.64×10^{-5}
390	2.18	0.0929	4.91	5.82×10^{-6}	0.093	3.26×10^{-5}
420	2.45	0.0497	5.18	1.49×10^{-6}	0.050	1.75×10^{-5}
450	2.73	0.0241	5.45	3.55×10^{-7}	0.024	8.40×10^{-6}

These values are plotted in Figure 7-4.

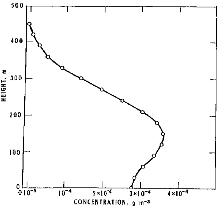

Figure 7-4. Concentration as a function of height (Problem 9).

Verifying:

$$\chi\,(x,0,0) = \frac{Q}{\pi\,\sigma_y\,\sigma_z\,u}\exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]$$

$$= \frac{151}{\pi\;181\;(136)\;4}\exp\left[-\frac{1}{2}\left(\frac{150}{136}\right)^2\right]$$

$$= 4.88 \times 10^{-4} \exp\left[-\tfrac{1}{2}\,(1.10)^2\right]$$

$$= 4.88 \times 10^{-4}\,(0.546)$$

$$= 2.7 \times 10^{-4}\ \text{g m}^{-3}$$

$$\chi\,(x,0,150) = \frac{Q}{2\pi\,\sigma_y\,\sigma_z\,u}\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right]\right\}$$

$$= \frac{151}{2\pi\;181\;(136)\;4}\left\{\exp\left[-\frac{1}{2}\left(\frac{0}{136}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{300}{136}\right)^2\right]\right\}$$

$$= 2.44 \times 10^{-4}\left\{1.0 + \exp\left[-\frac{1}{2}\,(2.21)^2\right]\right\}$$

$$= 2.44 \times 10^{-4}\,(1.0 + 8.70 \times 10^{-2})$$

$$= 2.44 \times 10^{-4}\,(1.087)$$

$$= 2.7 \times 10^{-4}\ \text{g m}^{-3}$$

PROBLEM 11: For the power plant in problem 4, what will the maximum ground-level concentration be beneath the plume centerline and at what distance will it occur on a clear night with wind speed 4 m sec^{-1}?

SOLUTION: A clear night with wind speed 4 m sec^{-1} indicates E stability conditions. From Figure 3-9, the maximum concentration should occur at a distance of 13 km, and the maximum $\chi u/Q$ is 1.7×10^{-6}

$$\chi_{max} = \frac{\chi u}{Q} \times \frac{Q}{u} = \frac{1.7 \times 10^{-6} \times 151}{4}$$

$$= 6.4 \times 10^{-5}\ \text{g m}^{-3}\ \text{of}\ SO_2$$

PROBLEM 12: For the situation in problem 11, what would the fumigation concentration be the next morning at this point (x = 13 km) when superadiabatic lapse rates extend to include most of the plume and it is assumed that wind speed and direction remain unchanged?

SOLUTION: The concentration during fumigation conditions is given by Eq. (5.2) with the exponential involving y equal to 1. in this problem.

$$\chi_F\,(x,0,0;H) = \frac{Q}{\sqrt{2\pi}\,u\,\sigma_{yF}\,h_i}$$

For the stable conditions, which were assumed to be class E, at x = 13 km, σ_y = 520 m., and σ_z = 90 m. Using Eq. (5.3) to solve for h_i:

$h_i = H + 2\,\sigma_z = 150 + 2\,(90) = 330$ m.

From the horizontal spreading suggested by Eq. (5.4):

$\sigma_{yF} = \sigma_y \text{ (stable)} + H/8 = 520 + 19 = 539$

$$\chi_F = \frac{151}{\sqrt{2\pi}\ 4\ (539)\ 330}$$

$= 8.5 \times 10^{-5}$ g m^{-3} of SO_2

Note that the fumigation concentrations under these conditions are about 1.3 times the maximum ground-level concentrations that occurred during the night (problem 11).

PROBLEM 13: An air sampling station is located at an azimuth of 203° from a cement plant at a distance of 1500 meters. The cement plant releases fine particulates (less than 15 microns diameter) at the rate of 750 pounds per hour from a 30-meter stack. What is the contribution from the cement plant to the total suspended particulate concentration at the sampling station when the wind is from 30° at 3 m sec^{-1} on a clear day in the late fall at 1600?

SOLUTION: For this season and time of day the C class stability should apply. Since the sampling station is off the plume axis, the x and y distances can be calculated:

$$x = 1500 \cos 7° = 1489$$

$$y = 1500 \sin 7° = 183$$

The source strength is:

$$Q = 750 \text{ lb hr}^{-1} \times 0.126 \frac{\text{g sec}^{-1}}{\text{lb hr}^{-1}} = 94.5 \text{ g sec}^{-1}$$

At this distance, 1489 m, for stability C, σ_y = 150 m, σ_z = 87. The contribution to the concentration can be calculated from Eq. (3.2):

$$\chi\ (x,y,0;H) = \frac{Q}{\pi\ \sigma_y\ \sigma_z\ u} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$$

$$\exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]$$

$$= \frac{94.5}{\pi\ 150\ (87)\ 3} \exp\left[-0.5\left(\frac{183}{150}\right)^2\right]$$

$$\exp\left[-0.5\left(\frac{30}{87}\right)^2\right]$$

$$= \frac{94.5}{1.23 \times 10^5} \quad \exp\ [-0.5\ (1.22)^2]$$

$$\exp\ [-0.5\ (0.345)^2]$$

$= 7.68 \times 10^{-4}\ (0.475)\ (0.943)$

$= 3.4 \times 10^{-4}$ g m^{-3}

PROBLEM 14: A proposed source is to emit 72 g sec^{-1} of SO_2 from a stack 30 meters high with a diameter of 1.5 meters. The effluent gases are emitted at a temperature of 250°F (394°K) with an exit velocity of 13 m sec^{-1}. Plot on log-log paper a graph of maximum ground-level concentration as a function of wind speed for stability classes B and D. Determine the critical wind speed for these stabilities, i.e., the wind speed that results in the highest concentrations. Assume that the design atmospheric pressure is 970 mb and the design ambient air temperature is 20°C (293°K).

SOLUTION: Using Holland's effective stack height equation:

$$\Delta H = \frac{v_s\ d}{u}\left[1.5 + 2.68 \times 10^{-3}\ p\ \frac{T_s - T_a}{T_s}\ d\right]$$

$$= \frac{13\ (1.5)}{u}\left[1.5 + 2.68 \times 10^{-3}\ (970)\right.$$

$$\left.\left(\frac{394 - 293}{394}\right)(1.5)\right]$$

$$= \frac{19.5}{u}\left[1.5 + 2.6\left(\frac{101}{394}\right) 1.5\right]$$

$$= \frac{19.5}{u}\ [1.5 + 2.6\ (0.256)\ 1.5]$$

$$= \frac{19.5}{u}\ [1.5 + 1.0]$$

$$= \frac{19.5\ (2.5)}{u}$$

$$= \frac{48.8}{u}$$

The effective stack heights for various wind speeds and stabilities are summarized in Table 7-5.

Table 7-5 EFFECTIVE STACK HEIGHTS (PROBLEM 14)

	Class D		Class B	
u, m sec^{-1}	ΔH, m	h + ΔH, m	1.15 ΔH, m	h + 1.15 ΔH, m
0.5	97.6	127.6	112.2	142.2
1.0	48.8	78.8	56.1	86.1
1.5	32.6	62.6	37.5	67.5
2	24.4	54.4	28.1	58.1
3	16.3	46.3	18.7	48.7
5	9.8	39.8	11.3	41.3
7	7.0	37.0	8.0	38.0
10	4.9	34.9		
20	2.4	32.4		

By use of the appropriate height, H, the maximum concentration for each wind speed and stability can be determined by obtaining the maximum $\chi u/Q$ as a function of H and stability from Figure 3-9 and multiplying by the appropriate Q/u. The computations are summarized in Table 7-6, and plotted in Figure 7-5.

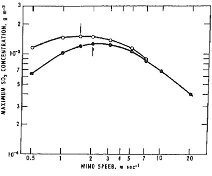

Figure 7-5. Maximum concentration as a function of wind speed (Problem 14).

Table 7-6 MAXIMUM CONCENTRATION AS A FUNCTION OF WIND SPEED (PROBLEM 14)

Stability Class	u, m sec^{-1}	H, m	$\chi u/Q_{max}$, m^{-2}	Q/u, g m^{-1}	χ_{max}, g m^{-3}
B	0.5	142.2	8.0×10^{-6}	144	1.15×10^{-3}
	1.0	86.1	2.0×10^{-5}	72	1.44×10^{-3}
	1.5	67.5	3.1×10^{-5}	48	1.49×10^{-3} ←
	2	58.1	4.1×10^{-5}	36	1.48×10^{-3}
	3	48.7	5.7×10^{-5}	24	1.37×10^{-3}
	5	41.3	7.8×10^{-5}	14.4	1.12×10^{-3}
	7	38.0	8.7×10^{-5}	10.3	8.96×10^{-4}
D	0.5	127.6	4.4×10^{-6}	144	6.34×10^{-4}
	1.0	78.8	1.42×10^{-5}	72	1.02×10^{-3}
	1.5	62.6	2.47×10^{-5}	48	1.19×10^{-3}
	2	54.4	3.5×10^{-5}	36	1.26×10^{-3} ←
	3	46.3	5.1×10^{-5}	24	1.22×10^{-3}
	5	39.8	7.3×10^{-5}	14.4	1.05×10^{-3}
	7	37.0	8.2×10^{-5}	10.3	8.45×10^{-4}
	10	34.9	9.4×10^{-5}	7.2	6.77×10^{-4}
	20	32.4	1.1×10^{-4}	3.6	3.96×10^{-4}

The wind speeds that give the highest maximum concentrations for each stability are, from Figure 7-5: B 1.5, D 2.0.

PROBLEM 15: A proposed pulp processing plant is expected to emit ½ ton per day of hydrogen sulfide from a single stack. The company property extends a minimum of 1500 meters from the proposed location. The nearest receptor is a small town of 500 inhabitants 1700 meters northeast of the plant. Plant managers have decided that it is desirable to maintain concentrations below 20 ppb (parts per billion by volume), or approximately 2.9×10^{-5} g m^{-3}, for any period greater than 30 minutes. Wind direction frequencies indicate that winds blow from the proposed location toward this town between 10 and 15 per cent of the time. What height stack should be erected? It is assumed that a design wind speed of 2 m sec^{-1} will be sufficient, since the effective stack rise will be quite great with winds less than 2 m sec^{-1}. Other than this stipulation, assume that the physical stack height and effective stack height are the same, to incorporate a slight safety factor.

SOLUTION: The source strength is:

$$Q = \frac{1000 \text{ lb day}^{-1} \times 453.6 \text{ g lb}^{-1}}{86{,}400 \text{ sec day}^{-1}} = 5.25 \text{ g sec}^{-1}$$

From Eq. (4.2):

$$\sigma_y \sigma_z = \frac{0.117\, Q}{\chi_d\, u} = \frac{0.117\ (5.25)}{(2.9 \times 10^{-5})\ 2}$$

$$= 1.06 \times 10^4 \text{ m}^2$$

At a design distance of 1500 meters (the limit of company property), $\sigma_y \sigma_z = 1.06 \times 10^4$ gives a point from Figure 4-1 about 0.2 from Class C to Class D along the line x = 1500 m. From Figure 3-3, $\sigma_z = 80$ for this stability.
$H = \sqrt{2}\, \sigma_z = 113$ meters

PROBLEM 16: In problem 15 assume that the stack diameter is to be 8 ft, the temperature of the effluent 250° F, and the stack gas velocity 45 ft sec^{-1}. From Holland's equation for effective stack height and the method used in problem 15, determine the physical stack height required to satisfy the conditions in problem 15. In estimating ΔH, use $T_a = 68°F$ and p = 920 mb.

SOLUTION: First determine the relation between ΔH and u from Holland's equation.

$v_s = 45$ ft $sec^{-1} = 13.7$ m sec^{-1}

d = 8 ft = 2.44 m

$T_s = 250°F = 121°C = 394°K$

$T_a = 68°F = 20°C = 293°K$

p = 920 mb

$$\Delta H = \frac{v_s\, d}{u}\left[1.5 + 2.68 \times 10^{-3}\, p\, \frac{T_s - T_a}{T_s}\, d\right]$$

$$= \frac{13.7\ (2.44)}{u}\left[1.5 + 2.68 \times 10^{-3}\ (920)\ \frac{394 - 293}{394}\ (2.44)\right]$$

$$= \frac{33.4}{u} [1.5 + (2.46)\ 0.256\ (2.44)]$$

$$= \frac{33.4}{u} (1.5 + 1.54)$$

$$\Delta H = \frac{102}{u}$$

The relation between $\sigma_y\ \sigma_z$ and u is:

$$\sigma_y\ \sigma_z = \frac{0.117\ Q}{\chi_d\ u} = \frac{0.117\ (5.25)}{2.9 \times 10^{-5}\ u} = \frac{2.12 \times 10^4}{u}$$

The required computations using Figure 4-1 are summarized in Table 7-7:

Table 7-7 REQUIREO PHYSICAL STACK HEIGHT AS A FUNCTION OF WIND SPEED (PROBLEM 16)

u, m sec⁻¹	ΔH, m	$\sigma_y\ \sigma_z$, m²	Stability to Give $\sigma_y\ \sigma_z$ at 1500 m	σ_z, m	$H' = \sqrt{2}\ \sigma_z$, m	$h = H' - \Delta H$, m
0.5	204	4.24×10^4	0.9 from A to B	190	269	65
1.0	102	2.12×10^4	0.6 from B to C	120	170	68
1.5	68	1.41×10^4	0.9 from B to C	96	136	68
2.0	51	1.06×10^4	0.2 from C to D	76	108	57
2.5	41	8.48×10^3	0.4 from C to D	64	91	50
3.0	34	7.06×10^3	0.6 from C to D	56	79	45
5.0	20	4.24×10^3	D	42	60	40
7.0	15	3.03×10^3	0.5 from D to E	34	48	33
10.0	10	2.12×10^3	E	28	40	30
15.0	7	1.41×10^3	0.5 from E to F	23	33	26

The required physical height is 68 meters.

PROBLEM 17: A dispersion study is being made over relatively open terrain with fluorescent particles whose size yields 1.8×10^{10} particles per gram of tracer. Sampling is by membrane filters through which 9×10^{-3} m³ of air is drawn each minute. A study involving a 1-hour release, which can be considered from ground-level, is to take place during conditions forecast to be slightly unstable with winds 5 m sec⁻¹. It is desirable to obtain a particle count of at least 20 particles upon membrane filters located at ground-level 2.0 km from the plume centerline on the sampling arc 8 km from the source. What should the total release be, in grams, for this run?

SOLUTION: The total dosage at the sampler is determined by the total sample in grams divided by the sampling rate:

$$D_T\ (\text{g sec m}^{-3}) = \frac{20 \text{ particles}}{1.8 \times 10^{10} \text{ particles g}^{-1}}$$

$$\frac{60 \text{ sec min}^{-1}}{9 \times 10^{-3} \text{ m}^3 \text{ min}^{-1}}$$

$$= \frac{1200}{16.2 \times 10^7}$$

$$D_T = 7.41 \times 10^{-6} \text{ g sec m}^{-3}$$

The total dosage is given in g sec m⁻³ from

$$D_T\ (x,y,0;0) = \frac{Q_T}{\pi\ u\ \sigma_y\ \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$$

where Q_T is the total release in grams.

$$\text{Therefore } Q_T = \frac{\pi\ u\ \sigma_y\ \sigma_z\ D_T}{\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]}$$

For slightly unstable conditions (Class C) at x = 8 km, σ_y = 690 m, σ_z = 310 m; y = 2000 m, u = 5 m sec⁻¹

$$Q_T = \frac{\pi\ 5\ (690)\ 310\ (7.41 \times 10^{-6})}{\exp\left[-\frac{1}{2}\left(\frac{2000}{690}\right)^2\right]}$$

$$= \frac{24.9}{\exp\ [-0.5\ (2.90)^2]}$$

$$= \frac{24.9}{1.49 \times 10^{-2}}$$

$$Q_T = 1670 \text{ g}$$

No correction has been made for the facts that the release is for 1 hour and the standard deviations represent time periods of 3 to 15 minutes.

PROBLEM 18: A release of 2 kg of fluorescent particles is made based on the results of the computation in problem 17. The conditions are class C stability and wind speed 5 m sec⁻¹. The crosswind-integrated ground-level dosage along the 8-km arc is determined from the samplers along this arc to be 8.2×10^{-1} g sec m⁻². What is the effective σ_z for this run?

SOLUTION: The crosswind-integrated dosage is given by:

$$D_{CWI} = \frac{2\ Q_T}{\sqrt{2\pi}\ \sigma_z\ u} \exp\left[-0.5\left(\frac{H}{\sigma_z}\right)^2\right]$$

Since the source is at ground-level, the exponential has a value of 1. Solving for σ_z:

$$\sigma_z = \frac{2\ Q_T}{\sqrt{2\pi}\ D_{CWI}\ u}$$

$$= \frac{2\ (2000)}{\sqrt{2\pi}\ (0.82)\ 5}$$

$$= \frac{4000}{10.28}$$

$$\sigma_z = 389 \text{ m}$$

PROBLEM 19: At a point directly downwind from a ground-level source the 3- to 15-minute concentration is estimated to be 3.4×10^{-3} g m^{-3}. What would you estimate the 2-hour concentration to be at this point, assuming no change in stability or wind velocity?

SOLUTION: Using Eq. (5.12) and letting k = 3 min, s = 2 hours, and p = 0.2:

$$\chi_{2\ hour} = \left(\frac{3}{120}\right)^{0.2} 3.4 \times 10^{-3}$$

$$= \frac{1}{40^{0.2}} (3.4 \times 10^{-3})$$

$$= \frac{3.4 \times 10^{-3}}{2.09} = 1.6 \times 10^{-3} \text{ g m}^{-3}$$

Letting k 15 min, s = 2 hours, and p = 0.17

$$\chi_{2\ hour} = \left(\frac{15}{120}\right)^{0.17} 3.4 \times 10^{-3}$$

$$= \frac{1}{8^{0.17}} (3.4 \times 10^{-3})$$

$$= \frac{3.4 \times 10^{-3}}{1.42} = 2.4 \times 10^{-3} \text{ g m}^{-3}$$

The 2-hour concentration is estimated to be between 1.6×10^{-3} and 2.4×10^{-3} g m^{-3}.

PROBLEM 20: Two sources of SO_2 are shown as points A and B in Figure 7-6. On a sunny summer afternoon the surface wind is from 60° at 6 m sec^{-1}. Source A is a power plant emitting 1450 g sec^{-1} SO_2 from two stacks whose physical height is 120 meters and whose ΔH, from Holland's equation, is ΔH (m) = 538 (m^2 sec^{-1})/u (m sec^{-1}). Source B is a refinery emitting 126 g sec^{-1} SO_2 from an effective height of 60 meters. The wind measured at 160 meters on a nearby TV tower is from 70° at 8.5 m sec^{-1}. Assuming that the mean direction of travel of both plumes is 245°, and there are no other sources of SO_2, what is the concentration of SO_2 at the receptor shown in the figure?

SOLUTION: Calculate the effective height of Source A using the observed wind speed at 160 meters.

$$\Delta H = \frac{538}{8.5} = 63.3$$

$H_A = 120 + 63 = 183$ m

$Q_A = 1450$ g sec^{-1}

$H_B = 60$ m

$Q_B = 126$ g sec^{-1}

For a sunny summer afternoon with wind speed 6 m sec^{-1}, the stability class to be expected is C. The equation to be used is Eq. (3.2):

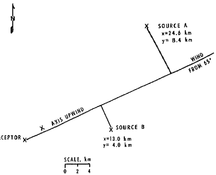

Figure 7-6. Locations of sources and receptor (Problem 20).

$$\chi(x,y,0;H) = \frac{Q}{\pi \sigma_y \sigma_z u} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]$$

For Source A, x = 24.6 km, y = 8.4 km

$\sigma_y = 1810$ m, $\sigma_z = 1120$ m, u = 8.5 m sec^{-1}

$$\chi_A = \frac{1450}{\pi\ 1810\ (1120)\ 8.5} \exp\left[-0.5 \left(\frac{8400}{1810}\right)^2\right] \exp\left[-0.5 \left(\frac{183}{1120}\right)^2\right]$$

$$= \frac{1450}{5.42 \times 10^7} \exp\left[-0.5\ (4.64)^2\right] \exp\left[-0.5\ (0.164)^2\right]$$

$$= 2.67 \times 10^{-5}\ (2.11 \times 10^{-5})\ (0.987)$$

$\chi_A = 5.6 \times 10^{-10}$ g m^{-3}

For Source B, x = 13.0 km, y = 4.0 km.

$\sigma_y = 1050$ m, $\sigma_z = 640$ m, u = 7.0 m sec^{-1}

$$\chi_B = \frac{126}{\pi\ 1050\ (640)\ 7} \exp\left[-0.5 \left(\frac{4000}{1050}\right)^2\right] \exp\left[-0.5 \left(\frac{60}{640}\right)^2\right]$$

$$= \frac{126}{1.48 \times 10^7} \exp\left[-0.5\ (3.81)^2\right] \exp\left[-0.5\ (0.0938)^2\right]$$

$$= 8.5 \times 10^{-6}\ (7.04 \times 10^{-4})\ (0.996)$$

$\chi_B = 6.0 \times 10^{-9}$ g m^{-3}

$\chi = \chi_A + \chi_B = 0.56 \times 10^{-9} + 6.0 \times 10^{-9}$

$= 6.6 \times 10^{-9}$ g m^{-3}

PROBLEM 21: A stack 15 meters high emits 3 g sec^{-1} of a particular air pollutant. The surrounding terrain is relatively flat except for a rounded hill about 3 km to the northeast whose crest extends 15 meters above the stack top. What is the highest 3- to 15-minute concentration of this pollutant that can be expected on the facing slope of the hill on a clear night when the wind is blowing directly from the stack toward the hill at 4 m sec^{-1}? Assume that ΔH is less than 15 m. How much does the wind have to shift so that concentrations at this point drop below 10^{-7} g m^{-3}?

SOLUTION: A clear night with 4 m sec^{-1} indicates class E stability. Eq. (3.4) for ground-level concentrations from a ground-level source is most applicable (See Chapter 5). At 3 km for class E, $\sigma_y = 140$ m, $\sigma_z = 43$ m.

$$\chi = \frac{Q}{\pi \sigma_y \sigma_z u} = \frac{3}{\pi\, 140\, (43)\, 4}$$

$\chi = 3.97 \times 10^{-5}$ g m^{-3}

To determine the crosswind distance from the plume centerline to produce a concentration of 10^{-7} g m^{-3} Eq. (3.8) is used:

$$y = \left[2 \ln \frac{\chi\,(x,0,0)}{\chi\,(x,y,0)} \right]^{1/2} \sigma_y$$

$$= \left[2 \ln \frac{3.97 \times 10^{-5}}{10^{-7}} \right]^{1/2} (140)$$

$$= (2 \ln 397)^{1/2}\, 140$$

$$= (2 \times 5.98)^{1/2}\, 140$$

$$= 3.46 \times 140$$

$$= 484 \text{ m.}$$

$$\tan \theta = \frac{484}{3000} = 0.1614$$

$$\theta = 9.2°$$

A wind shift of 9.2° is required to reduce the concentration to 10^{-7} g m^{-3}.

PROBLEM 22: An inventory of SO_2 emissions has been conducted in an urban area by square areas, 5000 ft (1524 meters) on a side. The emissions from one such area are estimated to be 6 g sec^{-1} for the entire area. This square is composed of residences and a few small commercial establishments. What is the concentration resulting from this area at the center of the adjacent square to the north when the wind is blowing from the south on a thinly overcast night with the wind at 2.5 m sec^{-1}? The average effective stack height of these sources is assumed to be 20 meters.

SOLUTION: A thinly overcast night with wind speed 2.5 m sec^{-1} indicates stability of class E. (It may actually be more unstable, since this is in a built-up area.) To allow for the area source, let $\sigma_{yo} = 1524/4.3 = 354$. For class E the virtual distance, $x_y = 8.5$ km. For x = 1524 m, $\sigma_z = 28.5$. For $x + x_y = 10{,}024$ m, $\sigma_y = 410$ m.

$$\chi = \frac{Q}{\pi \sigma_y \sigma_z u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2 \right]$$

$$= \frac{6}{\pi 410\, (28.5)\, 2.5} \exp\left[-\frac{1}{2}\left(\frac{20}{28.5}\right)^2 \right]$$

$$= 6.54 \times 10^{-5}\, (0.783)$$

$\chi = 5.1 \times 10^{-5}$ g m^{-3}

PROBLEM 23: An estimate is required of the total hydrocarbon concentration 300 meters downwind of an expressway at 1730 on an overcast day with wind speed 4 m sec^{-1}. The expressway runs north-south and the wind is from the west. The measured traffic flow is 8000 vehicles per hour during this rush hour, and the average speed of the vehicles is 40 miles per hour. At this speed the average vehicle is expected to emit 2×10^{-2} g sec^{-1} of total hydrocarbons.

SOLUTION: The expressway may be considered as a continuous infinite line source. To obtain a source strength q in grams sec^{-1} m^{-1}, the number of vehicles per meter of highway must be calculated and multiplied by the emission per vehicle.

$$\text{Vehicles/meter} = \frac{\text{Flow (vehicles hour}^{-1})}{\text{Average speed (miles hour}^{-1})\ 1600\ (\text{m mile}^{-1})}$$

$$= \frac{8000}{40 \times 1600} = 1.25 \times 10^{-1} \text{ (vehicles m}^{-1})$$

$q = 1.25 \times 10^{-1}$ (vehicles m^{-1}) $\times 2 \times 10^{-2}$ (g sec^{-1} $vehicle^{-1}$)

$q = 2.5 \times 10^{-3}$ (g sec^{-1} m^{-1})

Under overcast conditions with wind speed 4 m sec^{-1} stability class D applies. Under D, at x = 300 meters, $\sigma_z = 12$ m. From Eq. (5.18):

$$\chi\,(300,0,0;0) = \frac{2q}{\sqrt{2\pi}\,\sigma_z u}$$

$$= \frac{2\,(2.5 \times 10^{-3})}{2.507\,(12)\,4}$$

$= 4.2 \times 10^{-5}$ g m^{-3} of total hydrocarbons.

PROBLEM 24: A line of burning agricultural waste can be considered a finite line source 150 m long. It is estimated that the total emission of organics is at a rate of 90 g sec^{-1}. What is the 3- to 15-minute concentration of organics at a distance of 400 m directly downwind from the center of the line when the wind is blowing at 3 m sec^{-1} perpendicular to the line? Assume that it is 1600 on a sunny fall afternoon. What is the concentration directly downwind from one end of the source?

SOLUTION: Late afternoon at this time of year implies slight insolation, which with 3 m sec^{-1} winds yields stability class C. For C stability at x = 400 m, σ_y = 45 m, σ_z = 26 m.

$$q = \frac{Q}{150} = \frac{90}{150} = 0.6 \text{ g sec}^{-1} \text{ m}^{-1}$$

Eq. (5.20) is appropriate.

$$\chi\,(x,0,0;0) = \frac{2q}{\sqrt{2\pi}\,\sigma_z\,u} \int_{p_1}^{p_2} \frac{1}{\sqrt{2\pi}} \exp\,(-0.5\,p^2)\,dp$$

$$p_1 = \frac{y}{\sigma_y} = \frac{-75}{45} = -1.67,\ p_2 = \frac{y}{\sigma_y} = \frac{75}{45} = +1.67$$

$$\chi\,(400,0,0;0) = \frac{2\,(0.6)}{\sqrt{2\pi}(26)3} \int_{-1.67}^{+1.67} \frac{1}{\sqrt{2\pi}} \exp\,(-0.5\,p^2)\,dp$$

$= 6.14 \times 10^{-3}$ (0.91)

$= 5.6 \times 10^{-3}$ g m^{-3}

For a point downwind of one of the ends of the line:

$$p_1 = 0,\ p_2 = \frac{y}{\sigma_y} = \frac{150}{45} = +3.33$$

$$\chi\,(400,0,0;0) = 6.14 \times 10^{-3} \int_{0}^{+3.33} \frac{1}{\sqrt{2\pi}} \exp\,(-0.5\,p^2)\,dp$$

$= 6.14 \times 10^{-3}$ (0.4995)

$= 3.1 \times 10^{-3}$ g m^{-3}

PROBLEM 25: A core melt-down of a power reactor that has been operating for over a year occurs at 0200, releasing 1.5×10^6 curies of activity (1 second after the accident) into the atmosphere of the containment vessel. This total activity can be expected to decay according to $\left(\frac{t}{t_0}\right)^{-0.2}$. It is estimated that about 5.3×10^4 curies of this activity is due to iodine-131, which has a half-life of 8.04 days. The reactor building is hemispherically shaped with a radius of 20 meters. Assume the leak rate of the building is 0.1% day^{-1}. The accident has occurred on a relatively clear night with wind speed 2.5 m sec^{-1}. What is the concentration in the air 3 kilometers directly downwind from the source at 0400 due to all radioactive material? due to iodine-131?

SOLUTION: Source strength = leak rate x activity (corrected for decay)

$$\text{Leak rate} = \frac{0.001\ \text{day}^{-1}}{86400\ \text{sec day}^{-1}}$$

$= 1.157 \times 10^{-8}$ sec^{-1}

Source strength of all products

$$Q_A\ (\text{curies sec}^{-1}) = 1.157 \times 10^{-8}\ (1.5 \times 10^6) \left[\frac{t\ (\text{sec})}{t_0\ (\text{sec})}\right]^{-0.2}$$

$$= 1.74 \times 10^{-2} \left(\frac{t}{1}\right)^{-0.2}$$

To determine decay of materials with the half-life given, multiply by $\exp\left(\frac{-0.693\,t}{L}\right)$ where t is time and L is half-life.

Source strength of I^{131}.

$$Q_I\ (\text{curies sec}^{-1}) = 1.157 \times 10^{-8}\ (5.3 \times 10^4) \exp\left(\frac{-0.693\,t}{L}\right)$$

For I_{131} L = 6.95×10^5 sec

$$Q_I = 6.13 \times 10^{-4} \exp\left(\frac{-0.693\,t}{6.95 \times 10^5}\right)$$

For a clear night with wind speed 2.5 m sec^{-1}, class F applies. Approximate the spreading at the reactor shell by 2.15 σ_{yo} = 2.15 σ_{zo} = the radius of the shell = 20 m σ_{yo} = σ_{zo} = 9.3 m. The virtual distances to account for this are: x_y = 250 m, x_z = 560 m.

At x = 3000 m. x + x_y = 3250 m, σ_y = 100 m.
x + x_z = 3560 m, σ_z = 29 m.

$$\chi\,(x,0,0;0) = \frac{Q}{\pi\,\sigma_y\,\sigma_z\,u} = \frac{Q}{\pi\,100\,(29)\,2.5}$$

$= 4.4 \times 10^{-6}$ Q

For concentration at 0400, 3000 m downwind due to all radioactivity, t = 7200 seconds.

$\chi_A = 4.4 \times 10^{-6}\ (1.74 \times 10^{-2})\ (7200)^{-0.2}$
$= 7.66 \times 10^{-7}$ (0.17)

$\chi_A = 1.3 \times 10^{-7}$ curies m^{-3}

The concentration at 0400, 3000 m downwind due to I^{131} is:

$\chi_I = 4.4 \times 10^{-6}\ (6.13 \times 10^4) \exp\,[-0.997 \times 10^{-6}\ (7200)]$

$= 2.7 \times 10^{-8}$ (1.0) The decay of I^{131} is insignificant for 2 hours

$\chi_I = 2.7 \times 10^{-8}$ curies m^{-3}

PROBLEM 26: A spill estimated at 2.9×10^6 grams of unsymmetrical dimethyl hydrazine occurs at 0300 on a clear night while a rocket is being fueled. A circular area 60 meters in diameter built around the launch pad is revetted into squares 20 feet on a side to confine to as small an area as possible any spilled toxic liquids. In this spill only one such 20- by 20-foot area is involved. At the current wind speed of 2 m sec^{-1}, it is estimated that the evaporation rate will be 1100 g sec^{-1}. The wind direction is predicted to be from 310° ± 15° for the next hour. Table 7-8 gives the emergency tolerance limits for UDMH vapor.

Table 7-8 EMERGENCY TOLERANCE LIMITS FOR UDMH VAPOR VERSUS EXPOSURE TIME

Time, minutes	Emergency Tolerance Limits, g m^{-3}
5	1.2×10^{-1}
15	8.6×10^{-2}
30	4.9×10^{-2}
60	2.5×10^{-2}

What area should be evacuated?

SOLUTION: From Table 3-1, the stability class is determined to be Class F. This is not a point source but a small area source. Allowing 4.3 σ_{y0} to equal the width of the wetted area, 6.1 meters (20 feet), $\sigma_{y0} = 1.4$ meters. In attempting to determine the virtual distance, x_y, it is found to be less than 100 meters, and will be approximated by 40 meters. The release will take:

$$\frac{2.9 \times 10^6 \text{ g}}{1.1 \times 10^3 \text{ g sec}^{-1}} = 2.64 \times 10^3 \text{ sec} = 44 \text{ min.}$$

Therefore the concentration for an exposure time of 1 hour (2.5×10^{-2} g m^{-3}) is of main concern.

The equation for calculation of downwind concentrations is Eq. (3.4):

$$\chi\,(x,0;0) = \frac{Q}{\pi\, \sigma_y\, \sigma_z\, u}$$ where σ_y is a function of $x + x_y$.

Values of the parameters and of χ are given in Table 7-9.

Table 7-9 DETERMINATION OF CONCENTRATION AS A FUNCTION OF DISTANCE (PROBLEM 26)

x, km	σ_z, m	$x + x_y$, km	σ_y, m	χ, g m^{-3}
0.1	2.3	0.14	5.5	13.9
0.3	5.6	0.34	12.5	2.5
0.6	9.7	0.64	22	8.2×10^{-1}
1	14	1.04	35	3.6×10^{-1}
3	27	3.04	93	7.0×10^{-2}
6	37	6.04	175	2.7×10^{-2}
10	47	10.04	275	1.4×10^{-2}

These values of χ are graphed as a function of x in Figure 7-7. The downwind concentration drops below the critical value of 2.5×10^{-2} at a distance of 6.5 km.

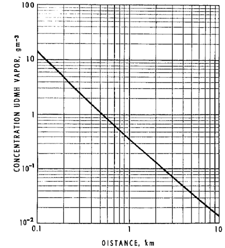

Figure 7-7. Concentration of UDMH as a function of downwind distance (Problem 26).

Calculated widths within a given isopleth are summarized in Table 7-10.

The maximum width of the area encompassed by an isopleth is about 140 meters from the downwind position. Since the wind direction is expected to be from 310°±15°, the sector at an azimuth of 115° to 145° plus a 140-meter rectangle on either side should be evacuated. See Figure 7-8.

Table 7-10 DETERMINATION OF WIDTHS WITHIN ISOPLETHS (PROBLEM 26)

x, km	$x + x_y$, km	σ_y, m	χ (centerline), g m^{-3}	$\frac{\chi \text{(isopleth)}}{\chi \text{(centerline)}}$	$\frac{y}{\sigma_y}$	y, m
0.1	0.14	5.5	13.9	1.8×10^{-3}	3.55	20
0.5	0.54	19	1.1	2.27×10^{-2}	2.75	52
1.0	1.04	35	3.6×10^{-1}	6.94×10^{-2}	2.31	80
2.0	2.04	66	1.3×10^{-1}	1.92×10^{-1}	1.82	120
3.0	3.04	93	7.0×10^{-2}	3.57×10^{-1}	1.44	134
4.0	4.04	120	4.8×10^{-2}	5.20×10^{-1}	1.14	137
5.0	5.04	149	3.5×10^{-2}	7.14×10^{-1}	0.82	122
6.0	6.04	175	2.7×10^{-2}	9.26×10^{-1}	0.39	68

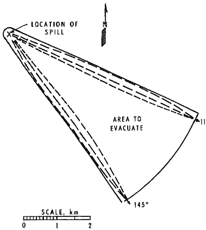

Figure 7-8. Possible positions of the 2.5×10^{-2} g m^{-3} isopleth and the evacuation area (Problem 26).

APPENDICES

Appendix 1: ABBREVIATIONS AND SYMBOLS

Abbreviations

- cal — calorie
- g — gram
- °K — degrees Kelvin
- m — meter
- mb — millibar
- sec — second

Symbols

- a — ratio of horizontal eddy velocity to vertical eddy velocity
- c_p — specific heat at constant pressure
- C_y — Sutton horizontal dispersion parameter
- C_z — Sutton vertical dispersion parameter
- d — inside stack diameter at stack top
- D_T (x,y,0;H) — Total dosage
- e — 2.7183, the base of natural logarithms
- f (θ,S,N) — frequency of wind direction for a given stability and wind speed class
- h — physical stack height
- h_i — height of the base of an inversion
- H — effective height of emission
- H_u — effective height of emission for a particular wind speed
- k — von Karman's constant, approximately equal to 0.4
- K — eddy diffusivity
- L — two uses: 1. the height of an air layer that is relatively stable compared to the layer beneath it; a lid
  2. the half-life of a radioactive material
- n — Sutton's exponent
- N — an index for wind speed class
- p — three uses: 1. Bosanquet's horizontal dispersion parameter
  2. atmospheric pressure
  3. a dummy variable in the equation for a Gaussian distribution.
- q — two uses: 1. Bosanquet's vertical dispersion parameter
  2. emission rate per length of a line source
- Q — emission rate of a source
- Q_T — total emission during an entire release
- R — net rate of sensible heating of an air column by solar radiation
- s — the length of the edge of a square area source
- S — an index for stability
- t_k — a short time period
- t_m — time required for the mixing layer to develop from the top of the stack to the top of the plume
- t_s — a time period
- T_a — ambient air temperature
- T_s — stack gas temperature at stack top
- u — wind speed
- u_N — a mean wind speed for the wind speed class N.
- v' — horizontal eddy velocity
- v_s — stack gas velocity at the stack top
- v_x — a velocity used by Calder
- w' — vertical eddy velocity
- x — distance downwind in the direction of the mean wind
- x_d — design distance, a particular downwind distance used for design purposes
- x_L — the distance at which $\sigma_z = 0.47L$
- x_x — a virtual distance so that $\sigma_x (x_x)$ equals the initial standard deviation, σ_{xo}
- x_y — a virtual distance so that $\sigma_y (x_y)$ equals the initial standard deviation, σ_{yo}
- x_z — a virtual distance so that $\sigma_z (x_z)$ equals the initial standard deviation, σ_{zo}
- y — crosswind distance
- z — height above ground level
- z_o — roughness parameter
- $\frac{\delta \theta}{\delta z}$ — the rate of change of potential temperature with height
- ΔH — the rise of the plume centerline above the stack top
- θ — two uses: 1. wind direction azimuth or sector
  2. potential temperature
- π — 3.1416
- ρ_A — ambient air density
- σ_A — the standard deviation of azimuth (wind direction) as determined from a wind vane or bi-directional vane
- σ_E — the standard deviation of wind elevation angle as determined from a bi-directional vane
- σ_x — the standard deviation in the downwind direction of a puff concentration distribution
- σ_{xo} — an initial downwind standard deviation
- σ_y — the standard deviation in the crosswind direction of the plume concentration distribution
- σ_{yo} — an initial crosswind standard deviation
- σ_z — the standard deviation in the vertical of the plume concentration distribution
- σ_{zL} — an effective σ_z equal to 0.8 L
- σ_{zo} — an initial vertical standard deviation
- σ_{zS} — the vertical standard deviation of the plume concentration at a particular downwind distance for the stability, S.

ϕ the angle between the wind direction and a line source

χ concentration

χ_{CWI} crosswind-integrated concentration

χ_d a ground-level concentration for design purposes

χ_F inversion break-up fumigation concentration

χ_k concentration measured over a sampling time, t_k

χ_{max} maximum ground-level centerline concentration with respect to downwind distance

χ_s concentration measured over a sampling time, t_s

$\frac{\chi}{Q}$ relative concentration

$\frac{\chi u}{Q}$ relative concentration normalized for wind speed

χ (x,y,z;H) concentration at the point (x, y, z) from an elevated source with effective height, H.

χ (x,Θ) the long-term average concentration at distance x, for a direction Θ from a source.

Appendix 2: CHARACTERISTICS OF THE GAUSSIAN DISTRIBUTION

The Gaussian or normal distribution can be depicted by the bellshaped curve shown in Figure A-1. The equation for the ordinate value of this curve is:

$$y = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{1}{2}\left(\frac{x - \bar{x}}{\sigma}\right)^2\right] \quad (A.1)$$

Figure A-2 gives the ordinate value at any distance from the center of the distribution (which occurs at $\bar{x}$). This information is also given in Table A-1. Figure A-3 gives the area under the Gaussian curve from $-\infty$ to a particular value of p where $p = \frac{x - \bar{x}}{\sigma}$.

This area is found from Eq. (A.2):

$$\text{Area } (-\infty \text{ to } p) = \int_{-\infty}^{p} \frac{1}{\sqrt{2\pi}} \exp(-0.5\, p^2)\, dp \quad (A.2)$$

Figure A-4 gives the area under the Gaussian curve from $-p$ to $+p$. This can be found from Eq. (A.3):

$$\text{Area } (-p \text{ to } +p) = \int_{-p}^{+p} \frac{1}{\sqrt{2\pi}} \exp(-0.5\, p^2)\, dp \quad (A.3)$$

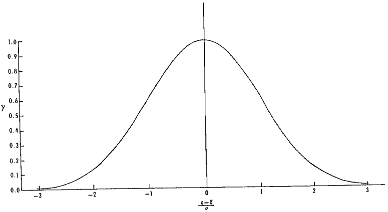

Figure A-1. The Gaussian distribution curve.

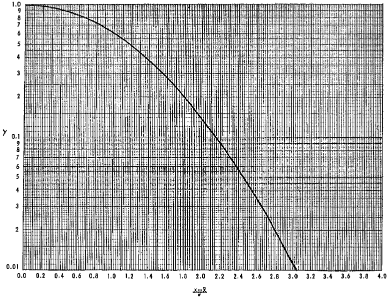

Figure A-2. Ordinate values of the Gaussian distribution.

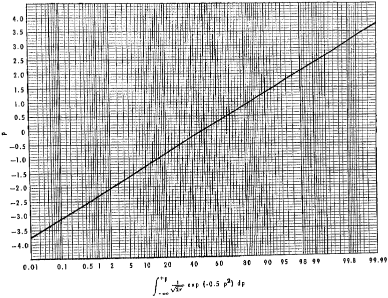

Figure A-3. Area under the Gaussian distribution curve from $-\infty$ to p.

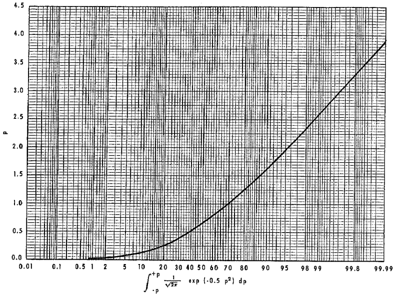

Figure A-4. Area under the Gaussian distribution curve between —p and +p.

Appendix 3: SOLUTIONS TO EXPONENTIALS

Expressions of the form exp $[-0.5\ A^2]$ where A is H/σ_z or y/σ_y frequently must be evaluated. Table A-1 gives B as a function of A where B = exp $[-0.5\ A^2]$. The sign and digits to the right of the E are to be considered as an exponent of 10. For example, if A is 3.51, B is given as 2.11E — 03 which means 2.11×10^{-3}

Table A-1 SOLUTIONS TO EXPONENTIALS B = exp [—0.5A²]
The notation 2.16 E-1 means 2.16 x 10⁻¹

A	0.00 B	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	1.00E 0	1.00E 0	10.00E -1	10.00E -1	9.99E -1	9.99E -1	9.98E -1	9.98E -1	9.97E -1	9.96E -1
0.10	9.95E -1	9.94E -1	9.93E -1	9.92E -1	9.90E -1	9.89E -1	9.87E -1	9.86E -1	9.84E -1	9.82E -1
0.20	9.80E -1	9.78E -1	9.76E -1	9.74E -1	9.72E -1	9.69E -1	9.67E -1	9.64E -1	9.62E -1	9.59E -1
0.30	9.56E -1	9.53E -1	9.50E -1	9.47E -1	9.44E -1	9.41E -1	9.37E -1	9.34E -1	9.30E -1	9.27E -1
0.40	9.23E -1	9.19E -1	9.16E -1	9.12E -1	9.08E -1	9.04E -1	9.00E -1	8.95E -1	8.91E -1	8.87E -1
0.50	8.83E -1	8.78E -1	8.74E -1	8.69E -1	8.64E -1	8.60E -1	8.55E -1	8.50E -1	8.45E -1	8.40E -1
0.60	8.35E -1	8.30E -1	8.25E -1	8.20E -1	8.15E -1	8.10E -1	8.04E -1	7.99E -1	7.94E -1	7.88E -1
0.70	7.83E -1	7.77E -1	7.72E -1	7.66E -1	7.61E -1	7.55E -1	7.49E -1	7.44E -1	7.38E -1	7.32E -1
0.80	7.26E -1	7.20E -1	7.15E -1	7.09E -1	7.03E -1	6.97E -1	6.91E -1	6.85E -1	6.79E -1	6.73E -1
0.90	6.67E -1	6.61E -1	6.55E -1	6.49E -1	6.43E -1	6.37E -1	6.31E -1	6.25E -1	6.19E -1	6.13E -1
1.00	6.07E -1	6.01E -1	5.94E -1	5.88E -1	5.82E -1	5.76E -1	5.70E -1	5.64E -1	5.58E -1	5.52E -1
1.10	5.46E -1	5.40E -1	5.34E -1	5.28E -1	5.22E -1	5.16E -1	5.10E -1	5.04E -1	4.99E -1	4.93E -1
1.20	4.87E -1	4.81E -1	4.75E -1	4.69E -1	4.64E -1	4.58E -1	4.52E -1	4.46E -1	4.41E -1	4.35E -1
1.30	4.30E -1	4.24E -1	4.18E -1	4.13E -1	4.08E -1	4.02E -1	3.97E -1	3.91E -1	3.86E -1	3.81E -1
1.40	3.75E -1	3.70E -1	3.65E -1	3.60E -1	3.55E -1	3.50E -1	3.45E -1	3.39E -1	3.35E -1	3.30E -1
1.50	3.25E -1	3.20E -1	3.15E -1	3.10E -1	3.06E -1	3.01E -1	2.96E -1	2.92E -1	2.87E -1	2.83E -1
1.60	2.78E -1	2.74E -1	2.69E -1	2.65E -1	2.61E -1	2.56E -1	2.52E -1	2.48E -1	2.44E -1	2.40E -1
1.70	2.36E -1	2.32E -1	2.28E -1	2.24E -1	2.20E -1	2.16E -1	2.13E -1	2.09E -1	2.05E -1	2.02E -1
1.80	1.98E -1	1.94E -1	1.91E -1	1.87E -1	1.84E -1	1.81E -1	1.77E -1	1.74E -1	1.71E -1	1.68E -1
1.90	1.65E -1	1.61E -1	1.58E -1	1.55E -1	1.52E -1	1.49E -1	1.47E -1	1.44E -1	1.41E -1	1.38E -1
2.00	1.35E -1	1.33E -1	1.30E -1	1.27E -1	1.25E -1	1.22E -1	1.20E -1	1.17E -1	1.15E -1	1.13E -1
2.10	1.10E -1	1.08E -1	1.06E -1	1.04E -1	1.01E -1	9.91E -2	9.70E -2	9.50E -2	9.29E -2	9.09E -2
2.20	8.89E -2	8.70E -2	8.51E -2	8.32E -2	8.14E -2	7.96E -2	7.78E -2	7.60E -2	7.43E -2	7.27E -2
2.30	7.10E -2	6.94E -2	6.78E -2	6.62E -2	6.47E -2	6.32E -2	6.17E -2	6.03E -2	5.89E -2	5.75E -2
2.40	5.61E -2	5.48E -2	5.35E -2	5.22E -2	5.10E -2	4.97E -2	4.85E -2	4.73E -2	4.62E -2	4.51E -2
2.50	4.39E -2	4.29E -2	4.18E -2	4.07E -2	3.97E -2	3.87E -2	3.78E -2	3.68E -2	3.59E -2	3.49E -2
2.60	3.41E -2	3.32E -2	3.23E -2	3.15E -2	3.07E -2	2.99E -2	2.91E -2	2.83E -2	2.76E -2	2.68E -2
2.70	2.61E -2	2.54E -2	2.47E -2	2.41E -2	2.34E -2	2.28E -2	2.22E -2	2.16E -2	2.10E -2	2.04E -2
2.80	1.98E -2	1.93E -2	1.88E -2	1.82E -2	1.77E -2	1.72E -2	1.67E -2	1.63E -2	1.58E -2	1.54E -2
2.90	1.49E -2	1.45E -2	1.41E -2	1.37E -2	1.33E -2	1.29E -2	1.25E -2	1.22E -2	1.18E -2	1.15E -2
3.00	1.11E -2	1.08E -2	1.05E -2	1.02E -2	9.85E -3	9.55E -3	9.26E -3	8.98E -3	8.71E -3	8.45E -3
3.10	8.19E -3	7.94E -3	7.70E -3	7.46E -3	7.23E -3	7.00E -3	6.79E -3	6.58E -3	6.37E -3	6.17E -3
3.20	5.98E -3	5.79E -3	5.60E -3	5.43E -3	5.25E -3	5.09E -3	4.92E -3	4.77E -3	4.61E -3	4.46E -3
3.30	4.32E -3	4.18E -3	4.04E -3	3.91E -3	3.78E -3	3.66E -3	3.54E -3	3.42E -3	3.31E -3	3.20E -3
3.40	3.09E -3	2.99E -3	2.89E -3	2.79E -3	2.69E -3	2.60E -3	2.51E -3	2.43E -3	2.35E -3	2.27E -3
3.50	2.19E -3	2.11E -3	2.04E -3	1.97E -3	1.90E -3	1.83E -3	1.77E -3	1.71E -3	1.65E -3	1.59E -3
3.60	1.53E -3	1.48E -3	1.43E -3	1.38E -3	1.33E -3	1.28E -3	1.23E -3	1.19E -3	1.15E -3	1.11E -3
3.70	1.07E -3	1.03E -3	9.89E -4	9.53E -4	9.18E -4	8.84E -4	8.51E -4	8.20E -4	7.89E -4	7.60E -4
3.80	7.32E -4	7.05E -4	6.78E -4	6.53E -4	6.28E -4	6.04E -4	5.82E -4	5.60E -4	5.38E -4	5.18E -4
3.90	4.98E -4	4.79E -4	4.61E -4	4.43E -4	4.26E -4	4.09E -4	3.93E -4	3.78E -4	3.63E -4	3.49E -4
4.00	3.36E -4	3.22E -4	3.10E -4	2.97E -4	2.86E -4	2.74E -4	2.63E -4	2.53E -4	2.43E -4	2.33E -4
4.10	2.24E -4	2.15E -4	2.06E -4	1.98E -4	1.90E -4	1.82E -4	1.75E -4	1.68E -4	1.61E -4	1.54E -4
4.20	1.48E -4	1.42E -4	1.36E -4	1.30E -4	1.25E -4	1.20E -4	1.15E -4	1.10E -4	1.05E -4	1.01E -4
4.30	9.66E -5	9.25E -5	8.86E -5	8.49E -5	8.13E -5	7.78E -5	7.45E -5	7.13E -5	6.83E -5	6.53E -5
4.40	6.25E -5	5.98E -5	5.72E -5	5.48E -5	5.24E -5	5.01E -5	4.79E -5	4.58E -5	4.38E -5	4.19E -5
4.50	4.01E -5	3.83E -5	3.66E -5	3.50E -5	3.34E -5	3.20E -5	3.05E -5	2.92E -5	2.79E -5	2.66E -5
4.60	2.54E -5	2.43E -5	2.32E -5	2.21E -5	2.11E -5	2.02E -5	1.93E -5	1.84E -5	1.75E -5	1.67E -5
4.70	1.60E -5	1.52E -5	1.45E -5	1.39E -5	1.32E -5	1.26E -5	1.20E -5	1.15E -5	1.09E -5	1.04E -5
4.80	9.93E -6	9.46E -6	9.02E -6	8.59E -6	8.19E -6	7.80E -6	7.43E -6	7.08E -6	6.74E -6	6.42E -6
4.90	[illegible]	5.82E -6	5.54E -6	5.28E -6	5.02E -6	4.78E -6	4.55E -6	4.33E -6	4.12E -6	3.9[illegible]

Table A-1 (continued) SOLUTIONS TO EXPONENTIALS

A \ B	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.00	3.73E -6	3.55E -6	3.37E -6	3.21E -6	3.05E -6	2.90E -6	2.76E -6	2.62E -6	2.49E -6	2.37E -6
5.10	2.25E -6	2.14E -6	2.03E -6	1.93E -6	1.83E -6	1.74E -6	1.65E -6	1.57E -6	1.49E -6	1.42E -6
5.20	1.34E -6	1.28E -6	1.21E -6	1.15E -6	1.09E -6	1.04E -6	9.82E -7	9.32E -7	8.84E -7	8.38E -7
5.30	7.95E -7	7.54E -7	7.15E -7	6.78E -7	6.43E -7	6.09E -7	5.77E -7	5.47E -7	5.19E -7	4.91E -7
5.40	4.66E -7	4.41E -7	4.18E -7	3.96E -7	3.75E -7	3.55E -7	3.36E -7	3.18E -7	3.01E -7	2.85E -7
5.50	2.70E -7	2.56E -7	2.42E -7	2.29E -7	2.17E -7	2.05E -7	1.94E -7	1.83E -7	1.73E -7	1.64E -7
5.60	1.55E -7	1.47E -7	1.39E -7	1.31E -7	1.24E -7	1.17E -7	1.11E -7	1.05E -7	9.87E -8	9.32E -8
5.70	8.81E -8	8.32E -8	7.86E -8	7.42E -8	7.01E -8	6.62E -8	6.25E -8	5.90E -8	5.57E -8	5.25E -8
5.80	4.96E -8	4.68E -8	4.41E -8	4.16E -8	3.93E -8	3.70E -8	3.49E -8	3.29E -8	3.11E -8	2.93E -8
5.90	2.76E -8	2.60E -8	2.45E -8	2.31E -8	2.18E -8	2.05E -8	1.94E -8	1.82E -8	1.72E -8	1.62E -8
6.00	1.52E -8	1.43E -8	1.35E -8	1.27E -8	1.20E -8	1.13E -8	1.06E -8	9.98E -9	9.39E -9	8.84E -9
6.10	8.32E -9	7.82E -9	7.36E -9	6.92E -9	6.51E -9	6.12E -9	5.76E -9	5.41E -9	5.09E -9	4.78E -9
6.20	4.50E -9	4.23E -9	3.97E -9	3.73E -9	3.51E -9	3.29E -9	3.09E -9	2.91E -9	2.73E -9	2.56E -9
6.30	2.41E -9	2.26E -9	2.12E -9	1.99E -9	1.87E -9	1.75E -9	1.65E -9	1.55E -9	1.45E -9	1.36E -9
6.40	1.28E -9	1.20E -9	1.12E -9	1.05E -9	9.87E-10	9.25E-10	8.67E-10	8.13E-10	7.62E-10	7.14E-10
6.50	6.69E-10	6.27E-10	5.88E-10	5.50E-10	5.16E-10	4.83E-10	4.52E-10	4.24E-10	3.97E-10	3.71E-10
6.60	3.48E-10	3.25E-10	3.05E-10	2.85E-10	2.67E-10	2.50E-10	2.34E-10	2.19E-10	2.04E-10	1.91E-10
6.70	1.79E-10	1.67E-10	1.56E-10	1.46E-10	1.37E-10	1.28E-10	1.19E-10	1.12E-10	1.04E-10	9.74E-11
6.80	9.10E-11	8.50E-11	7.94E-11	7.42E-11	6.93E-11	6.47E-11	6.04E-11	5.64E-11	5.27E-11	4.92E-11
6.90	4.59E-11	4.28E-11	4.00E-11	3.73E-11	3.48E-11	3.25E-11	3.03E-11	2.82E-11	2.63E-11	2.46E-11
7.00	2.29E-11	2.14E-11	1.99E-11	1.86E-11	1.73E-11	1.61E-11	1.50E-11	1.40E-11	1.30E-11	1.22E-11
7.10	1.13E-11	1.05E-11	9.81E-12	9.14E-12	8.51E-12	7.92E-12	7.38E-12	6.87E-12	6.39E-12	5.95E-12
7.20	5.54E-12	5.15E-12	4.79E-12	4.46E-12	4.15E-12	3.86E-12	3.59E-12	3.34E-12	3.10E-12	2.88E-12
7.30	2.68E-12	2.49E-12	2.32E-12	2.15E-12	2.00E-12	1.86E-12	1.73E-12	1.60E-12	1.49E-12	1.38E-12
7.40	1.29E-12	1.19E-12	1.11E-12	1.03E-12	9.55E-13	8.87E-13	8.23E-13	7.64E-13	7.09E-13	6.58E-13
7.50	6.10E-13	5.66E-13	5.25E-13	4.87E-13	4.52E-13	4.19E-13	3.88E-13	3.60E-13	3.34E-13	3.09E-13
7.60	2.87E-13	2.66E-13	2.46E-13	2.28E-13	2.11E-13	1.96E-13	1.81E-13	1.68E-13	1.56E-13	1.44E-13
7.70	1.34E-13	1.24E-13	1.14E-13	1.06E-13	9.80E-14	9.07E-14	8.39E-14	7.77E-14	7.19E-14	6.65E-14
7.80	6.15E-14	5.69E-14	5.26E-14	4.86E-14	4.50E-14	4.16E-14	3.84E-14	3.55E-14	3.28E-14	3.04E-14
7.90	2.80E-14	2.59E-14	2.39E-14	2.21E-14	2.04E-14	1.89E-14	1.74E-14	1.61E-14	1.49E-14	1.37E-14
8.00	1.27E-14	1.17E-14	1.08E-14	9.96E-15	9.19E-15	8.48E-15	7.82E-15	7.22E-15	6.66E-15	6.14E-15
8.10	5.66E-15	5.22E-15	4.81E-15	4.44E-15	4.09E-15	3.77E-15	3.48E-15	3.20E-15	2.95E-15	2.72E-15
8.20	2.51E-15	2.31E-15	2.13E-15	1.96E-15	1.80E-15	1.66E-15	1.53E-15	1.41E-15	1.30E-15	1.19E-15
8.30	1.10E-15	1.01E-15	9.30E-16	8.56E-16	7.87E-16	7.24E-16	6.66E-16	6.13E-16	5.64E-16	5.18E-16
8.40	4.77E-16	4.38E-16	4.03E-16	3.70E-16	3.40E-16	3.13E-16	2.87E-16	2.64E-16	2.43E-16	2.23E-16
8.50	2.05E-16	1.88E-16	1.73E-16	1.59E-16	1.46E-16	1.34E-16	1.23E-16	1.13E-16	1.03E-16	9.49E-17
8.60	8.71E-17	7.99E-17	7.33E-17	6.72E-17	6.17E-17	5.66E-17	5.19E-17	4.76E-17	4.36E-17	4.00E-17
8.70	3.67E-17	3.36E-17	3.08E-17	2.82E-17	2.59E-17	2.37E-17	2.17E-17	1.99E-17	1.82E-17	1.67E-17
8.80	1.53E-17	1.40E-17	1.28E-17	1.17E-17	1.07E-17	9.83E-18	9.00E-18	8.23E-18	7.53E-18	6.89E-18
8.90	6.31E-18	5.77E-18	5.29E-18	4.83E-18	4.41E-18	4.04E-18	3.69E-18	3.37E-18	3.08E-18	2.82E-18
9.00	2.58E-18	2.36E-18	2.15E-18	1.97E-18	1.80E-18	1.64E-18	1.50E-18	1.37E-18	1.25E-18	1.14E-18
9.10	1.04E-18	9.52E-19	8.69E-19	7.93E-19	7.24E-19	6.61E-19	6.03E-19	5.50E-19	5.02E-19	4.58E-19
9.20	4.18E-19	3.81E-19	3.47E-19	3.17E-19	2.89E-19	2.63E-19	2.40E-19	2.19E-19	1.99E-19	1.82E-19
9.30	1.66E-19	1.51E-19	1.37E-19	1.25E-19	1.14E-19	1.04E-19	9.46E-20	8.61E-20	7.84E-20	7.14E-20
9.40	6.50E-20	5.92E-20	5.38E-20	4.90E-20	4.46E-20	4.06E-20	3.69E-20	3.36E-20	3.05E-20	2.78E-20
9.50	2.53E-20	2.30E-20	2.09E-20	1.90E-20	1.73E-20	1.57E-20	1.43E-20	1.30E-20	1.18E-20	1.07E-20
9.60	9.72E-21	8.83E-21	8.02E-21	7.29E-21	6.62E-21	6.01E-21	5.46E-21	4.95E-21	4.50E-21	4.08E-21
9.70	3.70E-21	3.36E-21	3.05E-21	2.77E-21	2.51E-21	2.28E-21	2.07E-21	1.87E-21	1.70E-21	1.54E-21
9.80	1.40E-21	1.27E-21	1.15E-21	1.04E-21	9.43E-22	8.55E-22	7.75E-22	7.02E-22	6.36E-22	5.76E-22
9.90	5.22E-22	4.73E-22	4.28E-22	3.88E-22	3.51E-22	3.18E-22	2.88E-22	2.60E-22	2.36E-22	2.13E-22

Appendix 4: CONSTANTS, CONVERSION EQUATIONS, CONVERSION TABLES

Constants

$$e = 2.7183 \quad \frac{1}{e} = 0.3679$$

$$\pi = 3.1416 \quad \frac{1}{\pi} = 0.3183$$

$$2\pi = 6.2832 \quad \frac{1}{2\pi} = 0.1592$$

$$\sqrt{2\pi} = 2.5066 \quad \frac{1}{\sqrt{2\pi}} = 0.3989$$

$$\frac{2}{\sqrt{2\pi}} = 0.7979$$

$$(2\pi)^{3/2} = 15.75$$

Conversion Equations and Tables

$$T(°C) = 5/9\,(T(°F) - 32)$$

$$T(°K) = T(°C) + 273.16$$

$$T(°F) = (9/5\,T(°C)) + 32$$

CONVERSION FACTORS - VELOCITY

DESIRED UNITS / GIVEN UNITS	METERS PER SEC	FT PER SEC	FT PER MIN	KM PER HR	MI(STAT) PER HR	KNOTS	MI(STAT) PER DAY
METERS PER SEC	1.0000 E 00	3.2808 E 00	1.9685 E 02	3.6000 E 00	2.2369 E 00	1.9425 E 00	5.3686 E 01
FT PER SEC	3.0480 E-01	1.0000 E 00	6.0000 E 01	1.0973 E 00	6.8182 E-01	5.9209 E-01	1.6364 E 01
FT PER MIN	5.0800 E-03	1.6667 E-02	1.0000 E 00	1.8288 E-02	1.1364 E-02	9.8681 E-03	2.7273 E-01
KM PER HR	2.7778 E-01	9.1134 E-01	5.4681 E 01	1.0000 E 00	6.2137 E-01	5.3959 E-01	1.4913 E 01
MI(STAT) PER HR	4.4704 E-01	1.4667 E 00	8.8000 E 01	1.6093 E 00	1.0000 E 00	8.6839 E-01	2.4000 E 01
KNOTS	5.1479 E-01	1.6889 E 00	1.0134 E 02	1.8532 E 00	1.1516 E 00	1.0000 E 00	2.7637 E 01
MI(STAT) PER DAY	1.8627 E-02	6.1111 E-02	3.6667 E 00	6.7056 E-02	4.1667 E-02	3.6183 E-02	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT, MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNITS AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - EMISSION RATES

DESIRED UNITS / GIVEN UNITS	GRAMS PER SEC	GRAMS PER MIN	KG PER HOUR	KG PER DAY	LBS PER MIN	LBS PER HOUR	LBS PER DAY	TONS PER HOUR	TONS PER DAY
GRAMS PER SEC	1.0000 E 00	6.0000 E 01	3.6000 E 00	8.6400 E 01	1.3228 E-01	7.9366 E 00	1.9048 E 02	3.9683 E-03	9.5240 E-02
GRAMS PER MIN	1.6667 E-02	1.0000 E 00	6.0000 E-02	1.4400 E 00	2.2046 E-03	1.3228 E-01	3.1747 E 00	6.6139 E-05	1.5873 E-03
KG PER HOUR	2.7778 E-01	1.6667 E 01	1.0000 E 00	2.4000 E 01	3.6744 E-02	2.2046 E 00	5.2911 E 01	1.1023 E-03	2.6455 E-02
KG PER DAY	1.1574 E-02	6.9444 E-01	4.1667 E-02	1.0000 E 00	1.5310 E-03	9.1859 E-02	2.2046 E 00	4.5930 E-05	1.1023 E-03
LBS PER MIN	7.5599 E 00	4.5359 E 02	2.7216 E 01	6.5317 E 02	1.0000 E 00	6.0000 E 01	1.4400 E 03	3.0000 E-02	7.2000 E-01
LBS PER HOUR	1.2600 E-01	7.5599 E 00	4.5359 E-01	1.0886 E 01	1.6667 E-02	1.0000 E 00	2.4000 E 01	5.0000 E-04	1.2000 E-02
LBS PER DAY	5.2499 E-03	3.1499 E-01	1.8900 E-02	4.5359 E-01	6.9444 E-04	4.1667 E-02	1.0000 E 00	2.0833 E-05	5.0000 E-04
TONS PER HOUR	2.5200 E 02	1.5120 E 04	9.0718 E 02	2.1772 E 04	3.3333 E 01	2.0000 E 03	4.8000 E 04	1.0000 E 00	2.4000 E 01
TONS PER DAY	1.0500 E 01	6.2999 E 02	3.7799 E 01	9.0718 E 02	1.3889 E 00	8.3333 E 01	2.0000 E 03	4.1667 E-02	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT. MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNITS AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - LENGTH

DESIRED UNITS / GIVEN UNITS	METER	CM	MICRON	KILOMETER	INCH	FOOT	YARD	MILE(STAT)	MILE(NAUT)
METER	1.0000 E 00	1.0000 E 02	1.0000 E 06	1.0000 E-03	3.9370 E 01	3.2808 E 00	1.0936 E 00	6.2137 E-04	5.3959 E-04
CM	1.0000 E-02	1.0000 E 00	1.0000 E 04	1.0000 E-05	3.9370 E-01	3.2808 E-02	1.0936 E-02	6.2137 E-06	5.3959 E-06
MICRON	1.0000 E-06	1.0000 E-04	1.0000 E 00	1.0000 E-09	3.9370 E-05	3.2808 E-06	1.0936 E-06	6.2137 E-10	5.3959 E-10
KILOMETER	1.0000 E 03	1.0000 E 05	1.0000 E 09	1.0000 E 00	3.9370 E 04	3.2808 E 03	1.0936 E 03	6.2137 E-01	5.3959 E-01
INCH	2.5400 E-02	2.5400 E 00	2.5400 E 04	2.5400 E-05	1.0000 E 00	8.3333 E-02	2.7778 E-02	1.5783 E-05	1.3706 E-05
FOOT	3.0480 E-01	3.0480 E 01	3.0480 E 05	3.0480 E-04	1.2000 E 01	1.0000 E 00	3.3333 E-01	1.8939 E-04	1.6447 E-04
YARD	9.1440 E-01	9.1440 E 01	9.1440 E 05	9.1440 E-04	3.6000 E 01	3.0000 E 00	1.0000 E 00	5.6818 E-04	4.9340 E-04
MILE(STAT)	1.6093 E 03	1.6093 E 05	1.6093 E 09	1.6093 E 00	6.3360 E 04	5.2800 E 03	1.7600 E 03	1.0000 E 00	8.6839 E-01
MILE(NAUT)	1.8532 E 03	1.8532 E 05	1.8532 E 09	1.8532 E 00	7.2962 E 04	6.0802 E 03	2.0267 E 03	1.1516 E 00	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT. MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNITS AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - AREA

DESIRED UNITS / GIVEN UNITS	SQ METER	SQ KM	SQ CM	SQ INCH	SQ FOOT	SQ YARD	ACRE	SQ STAT MILE	SQ NAUT MILE
SQ METER	1.0000 E 00	1.0000 E-06	1.0000 E 04	1.5500 E 03	1.0764 E 01	1.1960 E 00	2.4710 E-04	3.8610 E-07	2.9116 E-07
SQ KM	1.0000 E 06	1.0000 E 00	1.0000 E 10	1.5500 E 09	1.0764 E 07	1.1960 E 06	2.4710 E 02	3.8610 E-01	2.9116 E-01
SQ CM	1.0000 E-04	1.0000 E-10	1.0000 E 00	1.5500 E-01	1.0764 E-03	1.1960 E-04	2.4710 E-08	3.8610 E-11	2.9116 E-11
SQ INCH	6.4516 E-04	6.4516 E-10	6.4516 E 00	1.0000 E 00	6.9444 E-03	7.7160 E-04	1.5942 E-07	2.4910 E-10	1.8785 E-10
SQ FOOT	9.2903 E-02	9.2903 E-08	9.2903 E 02	1.4400 E 02	1.0000 E 00	1.1111 E-01	2.2957 E-05	3.5870 E-08	2.7050 E-08
SQ YARD	8.3613 E-01	8.3613 E-07	8.3613 E 03	1.2960 E 03	9.0000 E 00	1.0000 E 00	2.0661 E-04	3.2283 E-07	2.4345 E-07
ACRE	4.0469 E 03	4.0469 E-03	4.0469 E 07	6.2726 E 06	4.3560 E 04	4.8400 E 03	1.0000 E 00	1.5625 E-03	1.1783 E-03
SQ STAT MILE	2.5900 E 06	2.5900 E 00	2.5900 E 10	4.0145 E 09	2.7878 E 07	3.0976 E 06	6.4000 E 02	1.0000 E 00	7.5411 E-01
SQ NAUT MILE	3.4345 E 06	3.4345 E 00	3.4345 E 10	5.3235 E 09	3.6969 E 07	4.1076 E 06	8.4869 E 02	1.3261 E 00	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT, MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNITS AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - VOLUME

GIVEN UNITS \ DESIRED UNITS	CU METER	LITER	CU INCH	CU FOOT	CU STAT MILE	CU NAUT MILE	U S FLUID OUNCE	U S QUART	U S GALLON
CU METER	1.0000 E 00	9.9997 E 02	6.1023 E 04	3.5314 E 01	2.3991 E-10	1.5711 E-10	3.3814 E 04	1.0567 E-03	2.6417 E 02
LITER	1.0000 E-03	1.0000 E 00	6.1025 E 01	3.5315 E-02	2.3992 E-13	1.5711 E-13	3.3815 E 01	1.0567 E-06	2.6418 E-01
CU INCH	1.6387 E-05	1.6387 E-02	1.0000 E 00	5.7870 E-04	3.9315 E-15	2.5746 E-15	5.5412 E-01	1.7316 E-08	4.3290 E-03
CU FOOT	2.8317 E-02	2.8316 E 01	1.7280 E 03	1.0000 E 00	6.7936 E-12	4.4488 E-12	9.5751 E 02	2.9922 E-05	7.4805 E 00
CU STAT MILE	4.1682 E 09	4.1681 E 12	2.5436 E 14	1.4720 E 11	1.0000 E 00	6.5486 E-01	1.4094 E 14	4.4045 E 06	1.1011 E 12
CU NAUT MILE	6.3650 E 09	6.3649 E 12	3.8842 E 14	2.2478 E 11	1.5270 E 00	1.0000 E 00	2.1523 E 14	6.7259 E 06	1.6815 E 12
U S FLUID OUNCE	2.9574 E-05	2.9573 E-02	1.8047 E 00	1.0444 E-03	7.0950 E-15	4.6462 E-15	1.0000 E 00	3.1250 E-08	7.8125 E-03
U S QUART	9.4635 E 02	9.4633 E 05	5.7750 E 07	3.3420 E 04	2.2704 E-07	1.4868 E-07	3.2000 E 07	1.0000 E 00	2.5000 E 05
U S GALLON	3.7854 E-03	3.7853 E 00	2.3100 E 02	1.3368 E-01	9.0817 E-13	5.9472 E-13	1.2800 E 02	4.0000 E-06	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT, MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNITS AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - MASS

DESIRED UNITS / GIVEN UNITS	GRAM	MICROGRAM	KILOGRAM	METRIC TON	SHORT TON	LONG TON	GRAIN	OUNCE (AVDP)	LB (AVDP)
GRAM	1.0000 E 00	1.0000 E 06	1.0000 E-03	1.0000 E-06	1.1023 E-06	9.8421 E-07	1.5432 E 01	3.5274 E-02	2.2046 E-03
MICROGRAM	1.0000 E-06	1.0000 E 00	1.0000 E-09	1.0000 E-12	1.1023 E-12	9.8421 E-13	1.5432 E-05	3.5274 E-08	2.2046 E-09
KILOGRAM	1.0000 E 03	1.0000 E 09	1.0000 E 00	1.0000 E-03	1.1023 E-03	9.8421 E-04	1.5432 E 04	3.5274 E 01	2.2046 E 00
METRIC TON	1.0000 E 06	1.0000 E 12	1.0000 E 03	1.0000 E 00	1.1023 E 00	9.8421 E-01	1.5432 E 07	3.5274 E 04	2.2046 E 03
SHORT TON	9.0718 E 05	9.0718 E 11	9.0718 E 02	9.0718 E-01	1.0000 E 00	8.9286 E-01	1.4000 E 07	3.2000 E 04	2.0000 E 03
LONG TON	1.0160 E 06	1.0160 E 12	1.0160 E 03	1.0160 E 00	1.1200 E 00	1.0000 E 00	1.5680 E 07	3.5840 E 04	2.2400 E 03
GRAIN	6.4799 E-02	6.4799 E 04	6.4799 E-05	6.4799 E-08	7.1428 E-08	6.3775 E-08	1.0000 E 00	2.2857 E-03	1.4286 E-04
OUNCE (AVDP)	2.8349 E 01	2.8349 E 07	2.8349 E-02	2.8349 E-05	3.1250 E-05	2.7902 E-05	4.3750 E 02	1.0000 E 00	6.2500 E-02
LB (AVDP)	4.5359 E 02	4.5359 E 08	4.5359 E-01	4.5359 E-04	5.0000 E-04	4.4643 E-04	7.0000 E 03	1.6000 E 01	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT, MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNITS AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - FLOW

GIVEN UNITS \ DESIRED UNITS	CU METER PER SEC	CU METER PER HR	LITER PER SEC	LITER PER MIN	LITER PER HR	CU FT PER SEC	CU FT PER MIN	CU FT PER HR	CU CM PER SEC
CU METER PER SEC	1.0000 E 00	3.6000 E 03	9.9997 E 02	5.9998 E 04	3.5999 E 06	3.5314 E 01	2.1189 E 03	1.2713 E 05	1.0000 E 06
CU METER PER HR	2.7778 E-04	1.0000 E 00	2.7777 E-01	1.6666 E 01	9.9997 E 02	9.8096 E-03	5.8857 E-01	3.5314 E 01	2.7778 E 02
LITER PER SEC	1.0000 E-03	3.6001 E 00	1.0000 E 00	6.0000 E 01	3.6000 E 03	3.5315 E-02	2.1189 E 00	1.2714 E 02	1.0000 E 03
LITER PER MIN	1.6667 E-05	6.0002 E-02	1.6667 E-02	1.0000 E 00	6.0000 E 01	5.8859 E-04	3.5315 E-02	2.1189 E 00	1.6667 E 01
LITER PER HR	2.7779 E-07	1.0000 E-03	2.7778 E-04	1.6667 E-02	1.0000 E 00	9.8098 E-06	5.8859 E-04	3.5315 E-02	2.7779 E-01
CU FT PER SEC	2.8317 E-02	1.0194 E 02	2.8316 E 01	1.6990 E 03	1.0194 E 05	1.0000 E 00	6.0000 E 01	3.6000 E 03	2.8317 E 04
CU FT PER MIN	4.7195 E-04	1.6990 E 00	4.7194 E-01	2.8316 E 01	1.6990 E 03	1.6667 E-02	1.0000 E 00	6.0000 E 01	4.7195 E 02
CU FT PER HR	7.8658 E-06	2.8317 E-02	7.8656 E-03	4.7194 E-01	2.8316 E 01	2.7778 E-04	1.6667 E-02	1.0000 E 00	7.8658 E 00
CU CM PER SEC	1.0000 E-06	3.6000 E-03	9.9997 E-04	5.9998 E-02	3.5999 E 00	3.5314 E-05	2.1189 E-03	1.2713 E-01	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT, MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNITS AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - CONCENTRATION, DENSITY

DESIRED UNITS / GIVEN UNITS	GRAM PER CU METER	MG PER CU METER	MICROGRAM PER CU M	MICROGRAM PER LITER	GRAIN PER CU FT	OUNCE PER CU FT	LB PER CU FT	GRAM PER CU FT	LB PER CU METER
GRAM PER CU METER	1.0000 E 00	1.0000 E 03	1.0000 E 06	1.0000 E 03	4.3700 E-01	9.9885 E-04	6.2428 E-05	2.8317 E-02	2.2046 E-03
MG PER CU METER	1.0000 E-03	1.0000 E 00	1.0000 E 03	1.0000 E 00	4.3700 E-04	9.9885 E-07	6.2428 E-08	2.8317 E-05	2.2046 E-06
MICROGRAM PER CU M	1.0000 E-06	1.0000 E-03	1.0000 E 00	1.0000 E-03	4.3700 E-07	9.9885 E-10	6.2428 E-11	2.8317 E-08	2.2046 E-09
MICROGRAM PER LITER	9.9997 E-04	9.9997 E-01	9.9997 E 02	1.0000 E 00	4.3699 E-04	9.9883 E-07	6.2427 E-08	2.8316 E-05	2.2046 E-06
GRAIN PER CU FT	2.2883 E 00	2.2883 E 03	2.2883 E 06	2.2884 E 03	1.0000 E 00	2.2857 E-03	1.4286 E-04	6.4799 E-02	5.0449 E-03
OUNCE PER CU FT	1.0011 E 03	1.0011 E 06	1.0011 E 09	1.0012 E 06	4.3750 E 02	1.0000 E 00	6.2500 E-02	2.8349 E 01	2.2072 E 00
LB PER CU FT	1.6018 E 04	1.6018 E 07	1.6018 E 10	1.6019 E 07	7.0000 E 03	1.6000 E 01	1.0000 E 00	4.5359 E 02	3.5314 E 01
GRAM PER CU FT	3.5314 E 01	3.5314 E 04	3.5314 E 07	3.5315 E 04	1.5432 E 01	3.5274 E-02	2.2046 E-03	1.0000 E 00	7.7855 E-02
LB PER CU METER	4.5359 E 02	4.5359 E 05	4.5359 E 08	4.5360 E 05	1.9822 E 02	4.5307 E-01	2.8317 E-02	1.2844 E 01	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT, MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNITS AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - DEPOSITION RATE (SHORT TON .STAT. MILE)

DESIRED UNITS / GIVEN UNITS	GM PER SQ M PER MO	KG PER SQ KM PER MO	MG PER SQ CM PER MO	TON PER SQ MI PER MO	OZ PER SQ FT PER MO	LB PER ACRE PERMO	GM PER SQ FT PER MO	MG PER SQ IN PER MO
GM PER SQ M PER MO	1.0000 E 00	1.0000 E 03	1.0000 E-01	2.8550 E 00	3.2771 E-03	8.9218 E 00	9.2903 E-02	6.4516 E-01
KG PER SQ KM PER MO	1.0000 E-03	1.0000 E 00	1.0000 E-04	2.8550 E-03	3.2771 E-06	8.9218 E-03	9.2903 E-05	6.4516 E-04
MG PER SQ CM PER MO	1.0000 E 01	1.0000 E 04	1.0000 E 00	2.8550 E 01	3.2771 E-02	8.9218 E 01	9.2903 E-01	6.4516 E 00
TON PER SQ MI PER MO	3.5026 E-01	3.5026 E 02	3.5026 E-02	1.0000 E 00	1.1478 E-03	3.1250 E 00	3.2541 E-02	2.2598 E-01
OZ PER SQ FT PER MO	3.0515 E 02	3.0515 E 05	3.0515 E 01	8.7120 E 02	1.0000 E 00	2.7225 E 03	2.8349 E 01	1.9687 E 02
LB PER ACRE PERMO	1.1208 E-01	1.1208 E 02	1.1208 E-02	3.2000 E-01	3.6731 E-04	1.0000 E 00	1.0413 E-02	7.2313 E-02
GM PER SQ FT PER MO	1.0764 E 01	1.0764 E 04	1.0764 E 00	3.0731 E 01	3.5274 E-02	9.6033 E 01	1.0000 E 00	6.9444 E 00
MG PER SQ IN PER MO	1.5500 E 00	1.5500 E 03	1.5500 E-01	4.4252 E 00	5.0795 E-03	1.3829 E 01	1.4400 E-01	1.0000 E 00

TO CONVERT A VALUE FROM A GIVEN UNIT TO A DESIRED UNIT, MULTIPLY THE GIVEN VALUE BY THE FACTOR OPPOSITE THE GIVEN UNIT! AND BENEATH THE DESIRED UNIT. NOTE THAT E-XX MEANS 10 TO THE -XX POWER.

CONVERSION FACTORS - PRESSURE

GIVEN UNITS \ DESIRED UNITS	MILLIBAR	BAR	ATMOSPHERE	DYNES PER SQ CM	KG PER SQ CM	LBS PER SQ IN	MM MERCURY	IN MERCURY
MILLIBAR	1.0000 E 00	1.0000 E-03	9.8692 E-04	1.0000 E 03	1.0197 E-03	1.4504 E-02	7.5006 E-01	2.9530 E-02
BAR	1.0000 E 03	1.0000 E 00	9.8692 E-01	1.0000 E 06	1.0197 E 00	1.4504 E 01	7.5006 E 02	2.9530 E 01
ATMOSPHERE	1.0133 E 03	1.0133 E 00	1.0000 E 00	1.0133 E 06	1.0332 E 00	1.4696 E 01	7.6000 E 02	2.9921 E 01
DYNES PER SQ CM	1.0000 E-03	1.0000 E-06	9.8692 E-07	1.0000 E 00	1.0197 E-06	1.4504 E-05	7.5006 E-04	2.9530 E-05
KG PER SQ CM	9.8066 E 02	9.8066 E-01	9.6784 E-01	9.8066 E 05	1.0000 E 00	1.4223 E 01	7.3556 E 02	2.8959 E 01
LBS PER SQ IN	6.8947 E 01	6.8947 E-02	6.8046 E-02	6.8947 E 04	7.0307 E-02	1.0000 E 00	5.1715 E 01	2.0360 E 00
MM MERCURY	1.3332 E 00	1.3332 E-03	1.3158 E-03	1.3332 E 03	1.3595 E-03	1.9337 E-02	1.0000 E 00	3.9370 E-02
IN MERCURY	3.3864 E 01	3.3864 E-02	3.3421 E-02	3.3864 E 04	3.4532 E-02	4.9115 E-01	2.5400 E 01	1.0000 E 00

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CONVERSION FACTORS - TIME

DESIRED UNITS / GIVEN UNITS	SECOND	MINUTE	HOUR	WEEK	MONTH (28)	MONTH (30)	MONTH (31)	YEAR (365)	YEAR (366)
SECOND	1.0000 E 00	6.0000 E 01	3.6000 E 03	6.0480 E 05	2.4192 E 06	2.5920 E 06	2.6784 E 06	3.1536 E 07	3.1622 E 07
MINUTE	1.6667 E-02	1.0000 E 00	6.0000 E 01	1.0080 E 04	4.0320 E 04	4.3200 E 04	4.4640 E 04	5.2560 E 05	5.2704 E 05
HOUR	2.7778 E-04	1.6667 E-02	1.0000 E 00	1.6800 E 02	6.7200 E 02	7.2000 E 02	7.4400 E 02	8.7600 E 03	8.7840 E 03
WEEK	1.6534 E-06	9.9206 E-05	5.9524 E-03	1.0000 E 00	4.0000 E 00	4.2857 E 00	4.4286 E 00	5.2143 E 01	5.2286 E 01
MONTH (28)	4.1336 E-07	2.4802 E-05	1.4881 E-03	2.5000 E-01	1.0000 E 00	1.0714 E 00	1.1071 E 00	1.3036 E 01	1.3071 E 01
MONTH (30)	3.8580 E-07	2.3148 E-05	1.3889 E-03	2.3333 E-01	9.3333 E-01	1.0000 E 00	1.0333 E 00	1.2167 E 01	1.2200 E 01
MONTH (31)	3.7336 E-07	2.2401 E-05	1.3441 E-03	2.2581 E-01	9.0323 E-01	9.6774 E-01	1.0000 E 00	1.1774 E 01	1.1806 E 01
YEAR (365)	3.1710 E-08	1.9026 E-06	1.1416 E-04	1.9178 E-02	7.6712 E-02	8.2192 E-02	8.4932 E-02	1.0000 E 00	1.0027 E 00
YEAR (366)	3.1623 E-08	1.8974 E-06	1.1384 E-04	1.9126 E-02	7.6503 E-02	8.1967 E-02	8.4699 E-02	9.9727 E-01	1.0000 E 00

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CONVERSION FACTORS - POWER

DESIRED UNITS / GIVEN UNITS	WATT (INT)	KILOWATT (INT)	MEGAWATT (INT)	CAL (INT) PER SEC	BTU PER MIN	BTU PER HR	JOULES ABS PER SEC	WATT (ABS)	ELECT. HORSEPOWER
WATT (INT)	1.0000 E 00	1.0000 E-03	1.0000 E-06	2.3880 E-01	5.6857 E-02	3.4114 E 00	1.0002 E 00	1.0002 E 00	1.3407 E-03
KILOWATT (INT)	1.0000 E 03	1.0000 E 00	1.0000 E-03	2.3880 E 02	5.6857 E 01	3.4114 E 03	1.0002 E 03	1.0002 E 03	1.3407 E 00
MEGAWATT (INT)	1.0000 E 06	1.0000 E 03	1.0000 E 00	2.3880 E 05	5.6857 E 04	3.4114 E 06	1.0002 E 06	1.0002 E 06	1.3407 E 03
CAL (INT) PER SEC	4.1876 E 00	4.1876 E-03	4.1876 E-06	1.0000 E 00	2.3810 E-01	1.4286 E 01	4.1884 E 00	4.1884 E 00	5.6145 E-03
BTU PER MIN	1.7588 E 01	1.7588 E-02	1.7588 E-05	4.2000 E 00	1.0000 E 00	6.0000 E 01	1.7591 E 01	1.7591 E 01	2.3581 E-02
BTU PER HR	2.9313 E-01	2.9313 E-04	2.9313 E-07	7.0000 E-02	1.6667 E-02	1.0000 E 00	2.9319 E-01	2.9319 E-01	3.9301 E-04
JOULES ABS PER SEC	9.9981 E-01	9.9981 E-04	9.9981 E-07	2.3875 E-01	5.6846 E-02	3.4108 E 00	1.0000 E 00	1.0000 E 00	1.3405 E-03
WATT (ABS)	9.9981 E-01	9.9981 E-04	9.9981 E-07	2.3875 E-01	5.6846 E-02	3.4108 E 00	1.0000 E 00	1.0000 E 00	1.3405 E-03
ELECT. HORSEPOWER	7.4586 E 02	7.4586 E-01	7.4586 E-04	1.7811 E 02	4.2407 E 01	2.5444 E 03	7.4600 E 02	7.4600 E 02	1.0000 E 00

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CONVERSION FACTORS - ENERGY, WORK

DESIRED UNITS / GIVEN UNITS	ERG	DYNE-CM	ABS JOULE	CAL (INT)	CAL (15)	INT KW-HR	ABS KW-HR	BTU
ERG	1.0000 E 00	1.0000 E 00	1.0000 E-07	2.3884 E-08	2.3892 E-08	2.7773 E-14	2.7778 E-14	9.4781 E-11
DYNE-CM	1.0000 E 00	1.0000 E 00	1.0000 E-07	2.3884 E-08	2.3892 E-08	2.7773 E-14	2.7778 E-14	9.4781 E-11
ABS JOULE	1.0000 E 07	1.0000 E 07	1.0000 E 00	2.3884 E-01	2.3892 E-01	2.7773 E-07	2.7778 E-07	9.4781 E-04
CAL (INT)	4.1868 E 07	4.1868 E 07	4.1868 E 00	1.0000 E 00	1.0003 E 00	1.1628 E-06	1.1630 E-06	3.9683 E-03
CAL (15)	4.1855 E 07	4.1855 E 07	4.1855 E 00	9.9968 E-01	1.0000 E 00	1.1624 E-06	1.1626 E-06	3.9671 E-03
INT KW-HR	3.6007 E 13	3.6007 E 13	3.6007 E 06	8.6000 E 05	8.6027 E 05	1.0000 E 00	1.0002 E 00	3.4128 E 03
ABS KW-HR	3.6000 E 13	3.6000 E 13	3.6000 E 06	8.5984 E 05	8.6011 E 05	9.9981 E-01	1.0000 E 00	3.4121 E 03
BTU	1.0551 E 10	1.0551 E 10	1.0551 E 03	2.5200 E 02	2.5208 E 02	2.9302 E-04	2.9307 E-04	1.0000 E 00

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CONVERSION FACTORS - ENERGY PER UNIT AREA

DESIRED UNITS / GIVEN UNITS	LANGLEY	CAL (15) PER SQ CM	BTU PER SQ FT	INT KW-HR PER SQ M	ABS JOULES PER SQ CM
LANGLEY	1.0000 E 00	1.0000 E 00	3.6855 E 00	1.1624 E-02	4.1855 E 00
CAL (15) PER SQ CM	1.0000 E 00	1.0000 E 00	3.6855 E 00	1.1624 E-02	4.1855 E 00
BTU PER SQ FT	2.7133 E-01	2.7133 E-01	1.0000 E 00	3.1540 E-03	1.1357 E 00
INT KW-HR PER SQ M	8.6029 E 01	8.6029 E 01	3.1706 E 02	1.0000 E 00	3.6007 E 02
ABS JOULES PER SQ CM	2.3892 E-01	2.3892 E-01	8.8054 E-01	2.7772 E-03	1.0000 E 00

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CONVERSION FACTORS - POWER PER UNIT AREA (CAL ARE 15 DEG)

DESIRED UNITS / GIVEN UNITS	CAL PER SQ M PER SEC	CAL PER SQ CM PER MIN	LANGLEY PER MIN	CAL PER SQ CM PER DAY	BTU PER SQ FT PER MIN	BTU PER SQ FT PER DAY	ABS WATT PER SQ CM
CAL PER SQ M PER SEC	1.0000 E 00	6.0000 E-03	6.0000 E-03	8.6400 E 00	2.2113 E-02	3.1843 E 01	4.1855 E-04
CAL PER SQ CM PER MIN	1.6667 E 02	1.0000 E 00	1.0000 E 00	1.4400 E 03	3.6855 E 00	5.3071 E 03	6.9758 E-02
LANGLEY PER MIN	1.6667 E 02	1.0000 E 00	1.0000 E 00	1.4400 E 03	3.6855 E 00	5.3071 E 03	6.9758 E-02
CAL PER SQ CM PER DAY	1.1574 E-01	6.9444 E-04	6.9444 E-04	1.0000 E 00	2.5594 E-03	3.6855 E 00	4.8443 E-05
BTU PER SQ FT PER MIN	4.5222 E 01	2.7133 E-01	2.7133 E-01	3.9072 E 02	1.0000 E 00	1.4400 E 03	1.8928 E-02
BTU PER SQ FT PER DAY	3.1404 E-02	1.8843 E-04	1.8843 E-04	2.7133 E-01	6.9445 E-04	1.0000 E 00	1.3144 E-05
ABS WATT PER SQ CM	2.3892 E 03	1.4335 E 01	1.4335 E 01	2.0643 E 04	5.2833 E 01	7.6079 E 04	1.0000 E 00

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